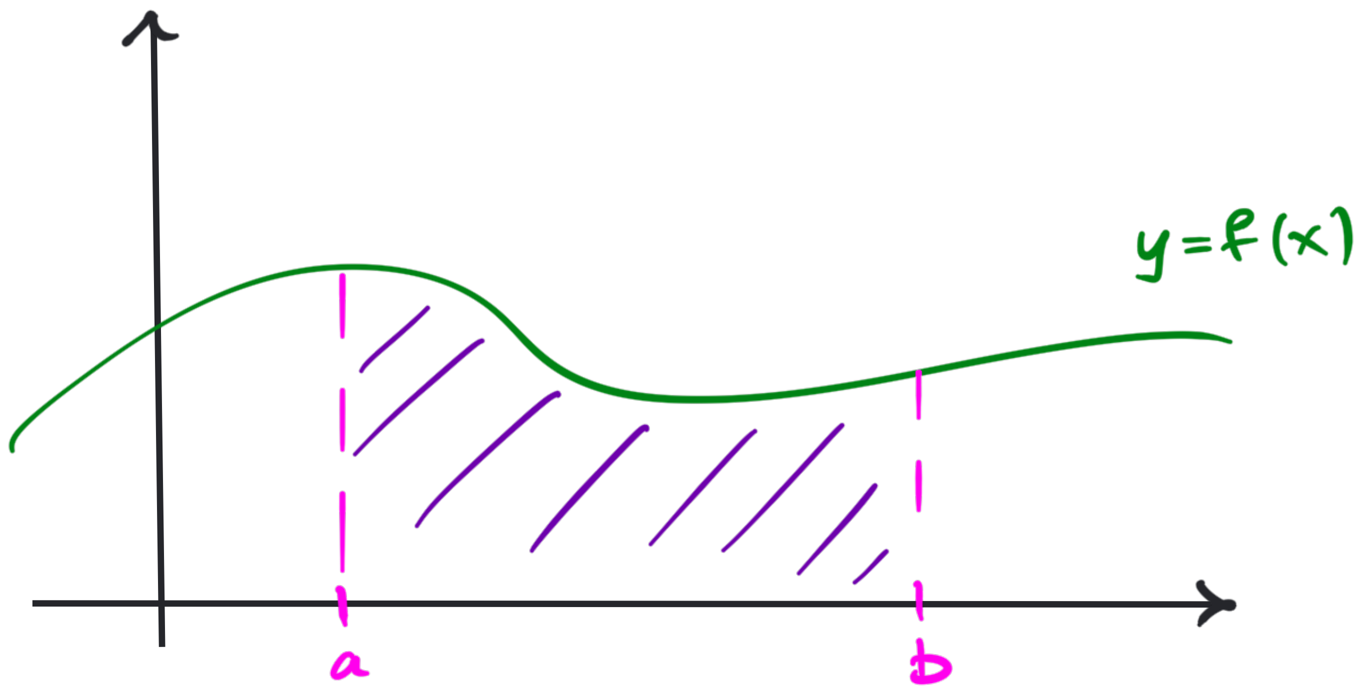
6. Numerical Methods

Goals: To be able to approximate definite integrals.



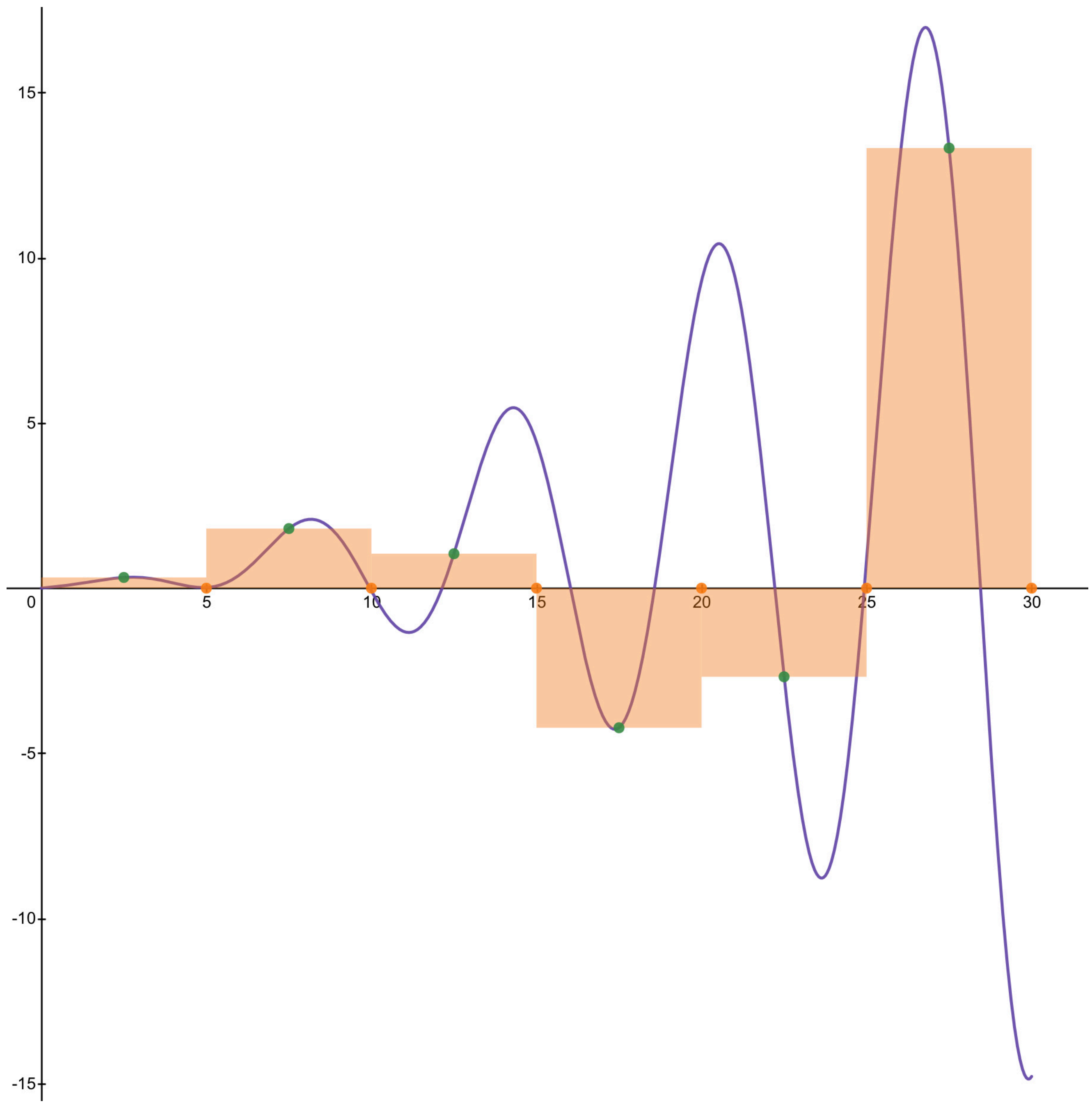
Remark: This is very useful because

(I) Midpoint Rule: For $n = 1, 2, 3, \ldots$ we approximate:

$$\boxed{\int_a^b f(x)\,dx \approx \Delta x \left(f\left(\frac{a+\Delta x}{2}\right) + f\left(\frac{a+2\Delta x}{2}\right) + \cdots + f\left(\frac{b-\Delta x}{2}\right)\right)}$$

where $\Delta x = \dfrac{b-a}{n}$.

Picture :



Notation : Mid(n) = Apply midpoint rule with n steps.

Observations :

1)

2)

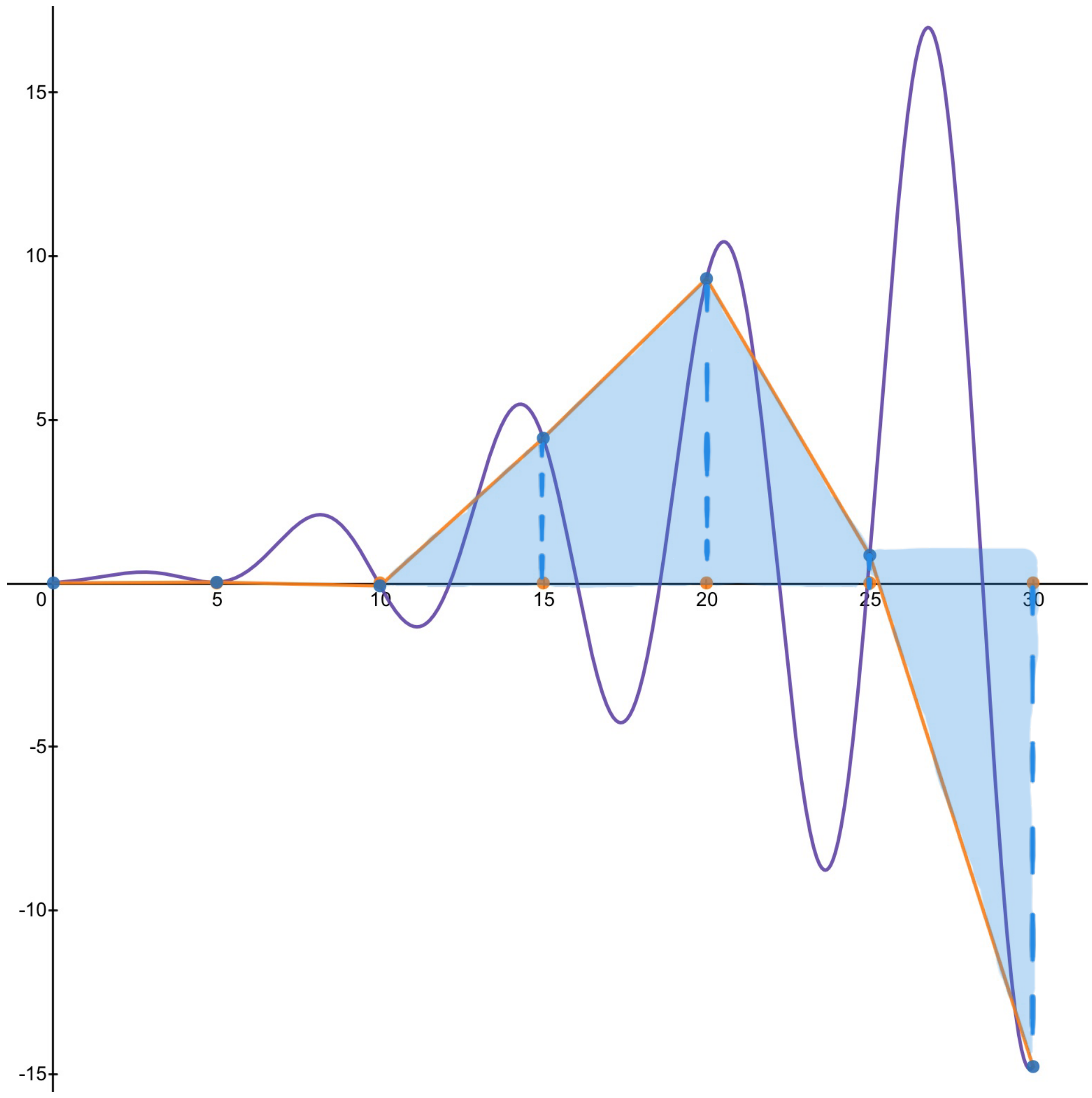
Example: Approximate $\int_1^2 \frac{1}{x^2}\,dx$ by Mid(3).

How close is it to the actual value?

(II) Trapezoidal Rule:

$$\boxed{\int_a^b f(x)\,dx \approx \frac{\Delta x}{2}\Big(f(a) + 2f(a+\Delta x) + \dots + 2f(b-\Delta x) + f(b)\Big)}$$

Picture:



Notation: Trap(n) = Trapezoidal Rule with n steps.

Obsevations:

1)

2)

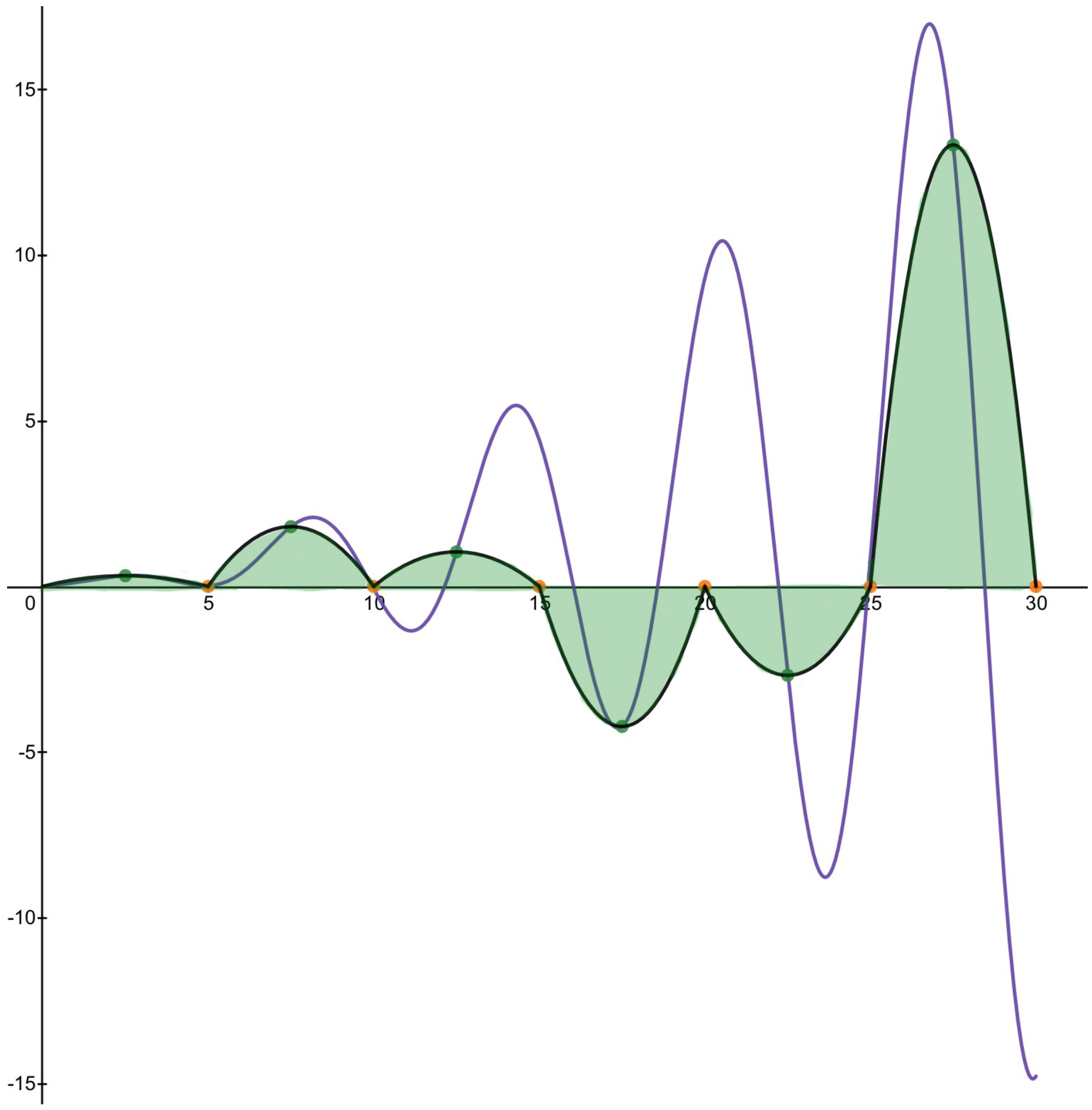
Example : Compute Trap(3) for $\int_1^2 \frac{1}{x^2}\,dx$.

Was it more/less accurate than Mid(3)?

(III) Simpson's Rule :

$$\int_a^b f(x)\,dx \approx \frac{\Delta x}{3}\Big(f(a) + 4f(a+\Delta x) + 2f(a+2\Delta x) + \dots$$
$$\dots + 2f(b-2\Delta x) + 4f(b-\Delta x) + f(b)\Big)$$

Picture :



Notation : Simp(n) = Simpson's Rule with n steps.

Observations :

1)

2)

Example: Compute Simp(6) for $\int_1^2 \frac{1}{x^2}\,dx$.

Exercises: Approximate $\int_0^6 x^2\,dx$ by:

1) Mid(3)

2) Trap(3)

3) Simp(4)