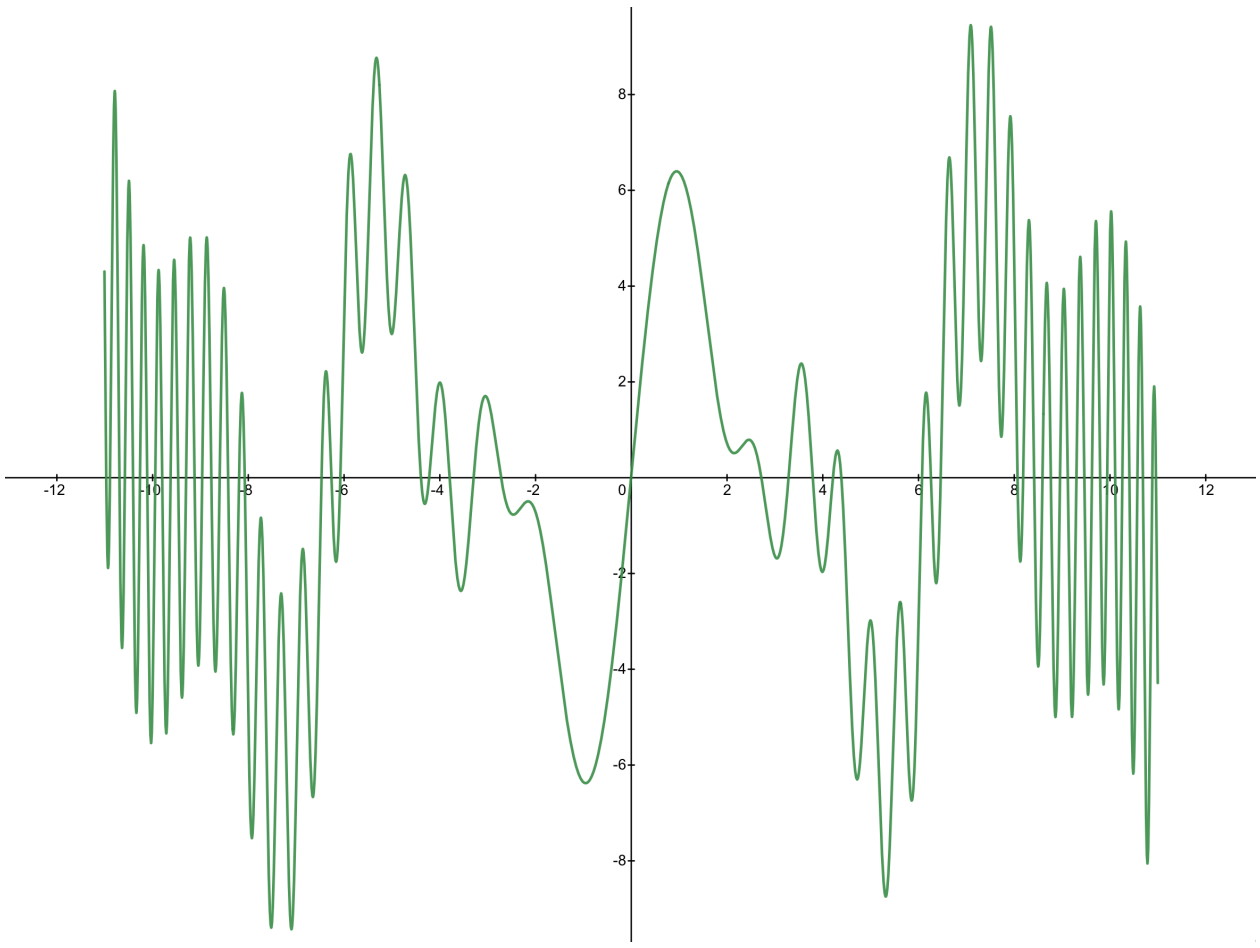
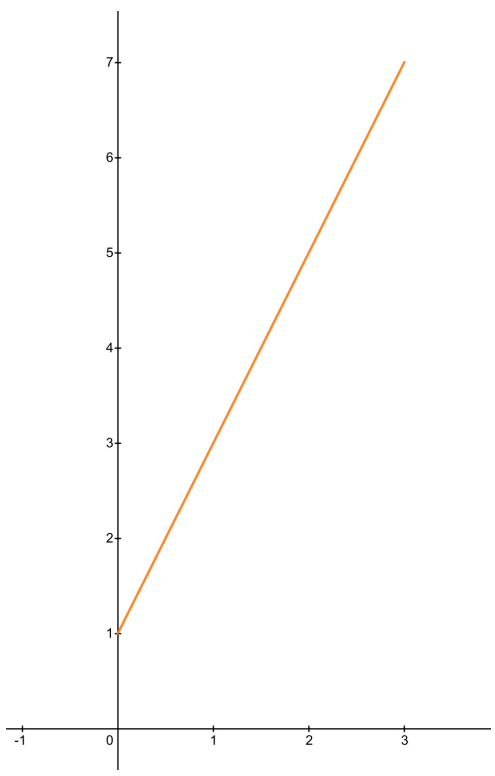
20. Arc Length

Goal : To figure out how to find lengths of of segments of certain curves.



Remark : The above example looks pretty difficult.

An easier example : $y = 2x + 1 \quad , \quad 0 \leq x \leq 3$



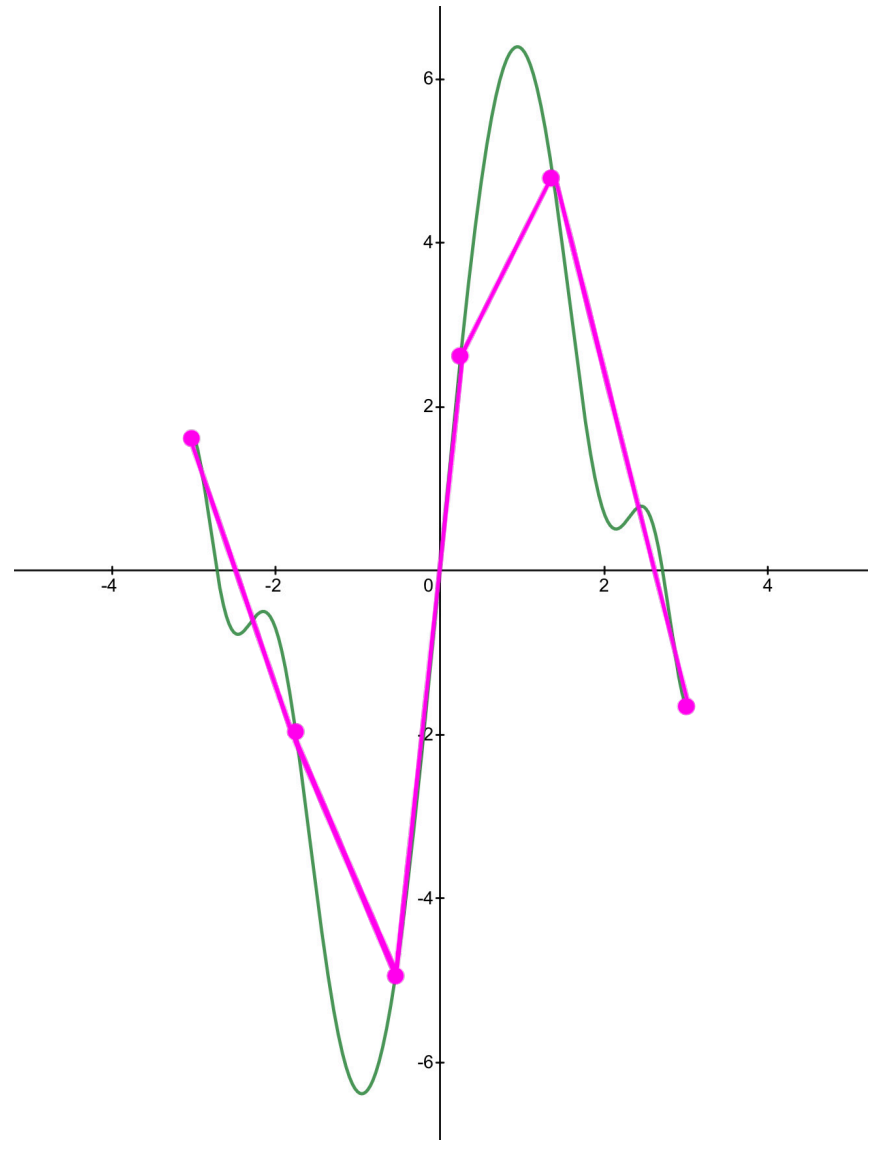
Remark :

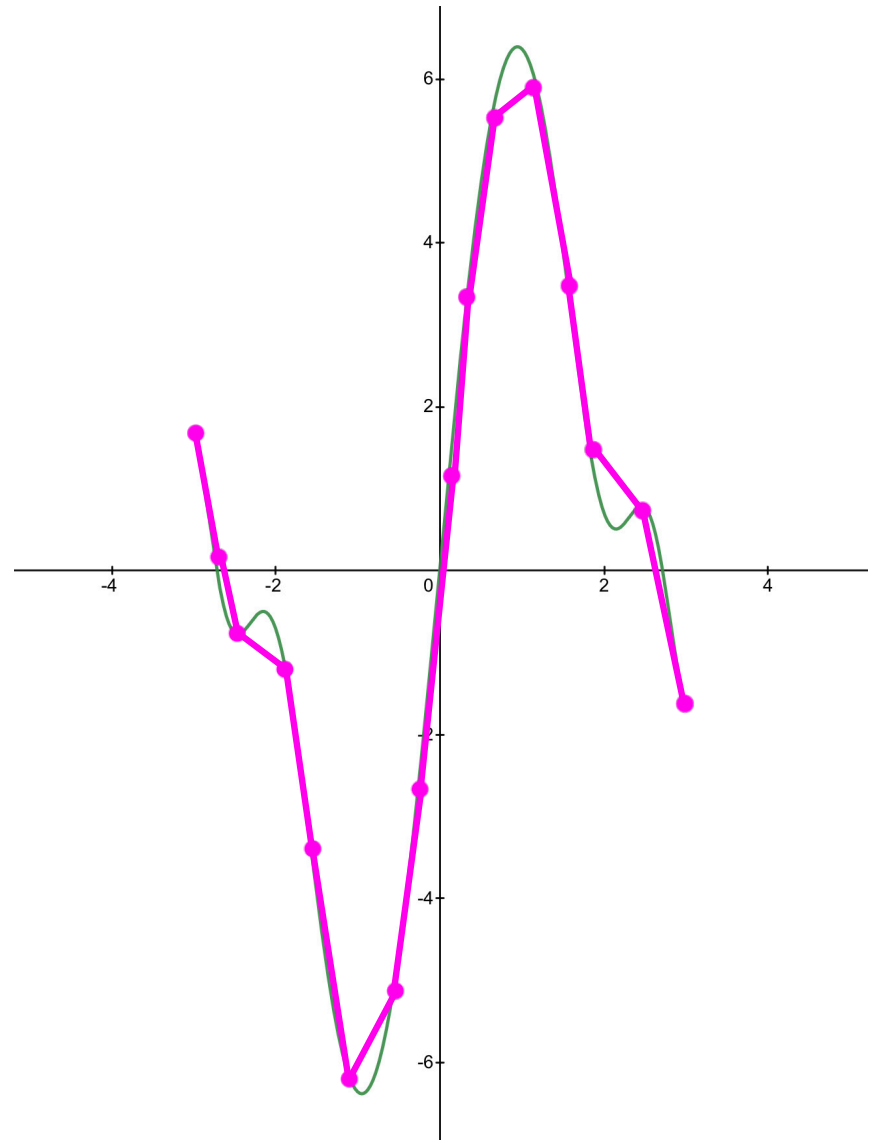
Idea :

Theorem : For a curve $y = f(x)$, the length of the segment from $(a, f(a))$ to $(b, f(b))$ is given by

$$L = \int_a^b \sqrt{1 + f'(x)^2}\, dx$$

Proof :


6
4
2
-2
-4
-6
-4
-2
0
2
4


6
4
2
-2
-4
-6
-4
-2
0
2
4

Example: Find the length of the curve segment given by $y = 3x^{3/2}$, $1 \leq x \leq 4$.

Example: Write down the integral which computes the length of the curve segment given by $y = \frac{x^2}{2}$ for $-\frac{1}{2} \leq x \leq 1/2$. Can you think of a way to approximate this integral?

Exercises: Find the length of the curve segment given by:

① $y = 2x - 5$, $-1 \leq x \leq 3$.

② $y = \sqrt{2 - x^2}$, $0 \leq x \leq 1$

③ $y = \sin x$, $0 \leq x \leq \pi$

④ $y = x - \ln x$, $1 \leq x \leq 4$

⑤ $y = xe^{-x}$, $0 \leq x \leq 2$