

2. Integration by Parts

Example: $\int x \sin x \, dx$

Problem:

We know how to differentiate a product

$$\frac{d}{dx}(uv) = \frac{du}{dx} \cdot v + u \cdot \frac{dv}{dx}$$

OR

$$(f \cdot g)'(x) = f'(x)g(x) + f(x) \cdot g'(x)$$

But what about integrating them?

Idea :

Result: Our formula for integration by parts:

$$\int f(x)g'(x)dx = f(x)g(x) - \int f'(x)g(x)dx$$

OR

$$\int u dv = uv - \int v du$$

Remark: We can use this for integrating awkward products. We just have to decide which part to integrate and which part to differentiate.

Example: $\int x \sin x \, dx$

Tip:

Examples:

1) $\int x^2 \cos x \, dx$

2) $\int \ln(x) \, dx$

3) $\int 3x^2 e^{x^3} \, dx \leftarrow \text{Careful.}$

Example : $\int e^x \sin x dx$

Problem :

Solution :

Moral :

Example : $\int e^x \cos x \, dx$

Definite Integrals :

$$\int_a^b f'(x) g(x) \, dx = f(x) g(x) \Big|_a^b - \int_a^b f(x) g'(x) \, dx$$

Examples:

$$1) \int_0^1 x^2 e^x dx$$

$$2) \int_0^1 \tan^{-1}(x) dx$$

Exercises:

1) $\int x e^x dx$

2) $\int t e^{st} dt$

3) $\int (w+1) e^{2w} dw$

4) $\int x^3 \ln x dx$

5) $\int \frac{z}{e^z} dz$

6) $\int \frac{\ln x}{x^2} dx$

7) (Harder) $\int e^{\sqrt{x}} dx$

Hint: u-sub, then I.B.P.