19. Population Growth and Decay:

Natural growth & decay is modeled by the differential equation:

$$\boxed{\frac{dP}{dt} = kP}$$

Intuition:

Remark: If $k > 0$, this is called the law of natural growth.

If $k < 0$, this is called the law of natural decay.

Solving this gives:

$$\boxed{P(t) = P_0 e^{kt}}$$

where P_0 = initial population.

Alternatively, we can write this as:

$$\boxed{P(t) = P_0 C^t}$$

Example: The population of a bacteria culture rises by 10% every hour. $P(0) = 2000$.

a. Find an expression for $P(t)$.

b. Find the population of the culture after 24 hours.

Example: The population of CalcLand was 700 on 01/01/2000 and was 3000 on 01/01/2010.

Using a model of exponential growth, find an estimate for the population on 01/01/2015.

Example : (Radioactive Decay)

The half life of Carbon-14 is approximately 5730 years.

A bowl made of oak has 40% of the carbon that it was crafted with.

Estimate the age of the bowl.

Application : Investments

If I invest $\$A_0$ in an account with an interest rate of $r \times 100\ \%$ per annum, compounded n times per year, the amount in the bank account after t years is given by :

$$\boxed{A(t) = A_0\left(1 + \frac{r}{n}\right)^{nt}}$$

Question: What happens with higher n?

If I invest $\$A_0$ in an account with an interest rate of $r \times 100\,\%$ per annum, compounded continuously, the amount in the bank account after t years is given by:

$$\boxed{A(t) = A_0 e^{rt}}$$

Example: How much will \$1000 amount to if invested in an account which compounds continuously at a rate of 4% per annum for 5 years?

Exercises:

1) You begin an experiment at 9am with a sample of 1000 bacteria. An hour later your population has doubled. What is the population at noon?

2) A bacteria culture contains 200 cells initially and grows at a rate proportional to its size. After 2 hours the population has reached 400. When will the population reach 5000?

3) Let $C(t)$ be the concentration of a drug in the bloodstream. As the body eliminates the drug, $C(t)$ decreases at a rate proportional to the amount of the drug present at that time.
$C(0) = 4\,mg/ml$. $C(5) = 3\,mg/ml$.
Find $C(t)$ and $C(10)$.