Name: ____________________

Instructor: ____________________

Math 10550, Practice Final Exam, December

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 2 hours.
- Be sure that your name is on this page.
- Be sure that you have all 25 problems.
- This is the only page you need to hand in.

Please mark your answers with an **X**! Do NOT circle them!

1.	a	b	c	d	e	15.	a	b	c	d	e
2.	a	b	c	d	e	16.	a	b	c	d	e
3.	a	b	c	d	e	17.	a	b	c	d	e
4.	a	b	c	d	e	18.	a	b	c	d	e
5.	a	b	c	d	e	19.	a	b	c	d	e
6.	a	b	c	d	e	20.	a	b	c	d	e
7.	a	b	c	d	e	21.	a	b	c	d	e
8.	a	b	c	d	e	22.	a	b	c	d	e
9.	a	b	c	d	e	23.	a	b	c	d	e
10.	a	b	c	d	e	24.	a	b	c	d	e
11.	a	b	c	d	e	25.	a	b	c	d	e
12.	a	b	c	d	e						
13.	a	b	c	d	e						
14.	a	b	c	d	e						

Final Exam: ____________

Previous Total: ____________

Course Total: ____________

1.(6 pts.) Compute $\lim_{x\to 2^-} \frac{x^2-4}{x^2-5x+6}$.

(a) -4 (b) ∞ (c) 0 (d) 1 (e) $-\infty$

$$\frac{x^2-4}{x^2-5x+6} = \frac{(x+2)(x-2)}{(x-3)(x-2)} = \frac{x+2}{x-3}$$

$$\Rightarrow \lim_{x\to 2^-} \frac{x^2-4}{x^2-5x+6} = \lim_{x\to 2^-} \frac{x+2}{x-3} = \frac{4}{-1} = -4$$

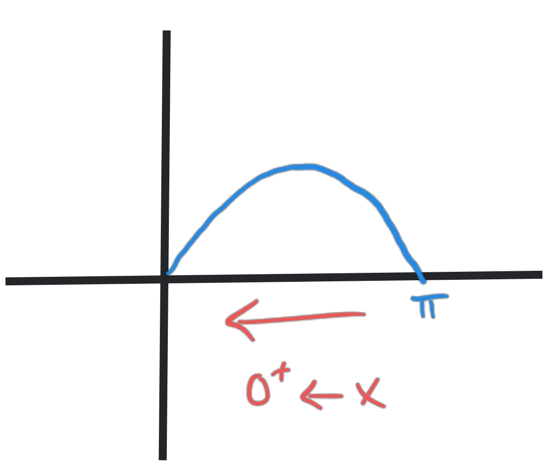
2.(6 pts.) Compute $\lim_{x\to 0^+} \frac{x^2-9}{\sin x}$.

(a) Does not exist and is not ∞ or $-\infty$. (b) ∞

(c) 0 (d) -9

(e) $-\infty$

$\lim_{x\to 0^+} x^2-9 = -9$ ← negative and finite.



$\lim_{x\to 0^+} \sin(x) = 0$ ← approaches from above i.e. eventually always positive.

$$\Rightarrow \lim_{x\to 0^+} \frac{x^2-9}{\sin(x)} = -\infty$$

$$\overbrace{\left(\sqrt{x^2-x}-\sqrt{x^2+5x}\right)}^{f(x)}$$

3.(6 pts.) Evaluate $\lim_{x\to\infty}\left(\sqrt{x^2-x}-\sqrt{x^2+5x}\,\right)$.

(a) 0 (b) 3 (c) -6

(d) Does not exist (e) -3

$$\sqrt{x^2-x}-\sqrt{x^2+5x} = \frac{\left(\sqrt{x^2-x}-\sqrt{x^2+5x}\right)\left(\sqrt{x^2-x}+\sqrt{x^2+5x}\right)}{\sqrt{x^2-x}+\sqrt{x^2+5x}}$$

$$= \frac{(x^2-x)-(x^2+5x)}{\sqrt{x^2-x}+\sqrt{x^2+5x}} = \frac{-6x}{\sqrt{x^2-x}+\sqrt{x^2+5x}} = \frac{-6}{\sqrt{1-\frac{1}{x}}+\sqrt{1+\frac{5}{x}}}$$

$$\Rightarrow \lim_{x\to\infty} f(x) = \frac{-6}{2} = -3$$

4.(6 pts.) For what constant a is the function f given by

$$f(x)=\begin{cases} ax+1 & x<0 \\ x^2+1 & x\geq 0\end{cases}$$

differentiable everywhere?

(a) $a=2$ (b) $a=0$

(c) $a=1$ (d) Any value of a

(e) No value of a

Clearly differentiable for $x\neq 0$: $f'(x)=\begin{cases} a & x<0 \\ 2x & x>0\end{cases}$

$$\lim_{h\to 0}\frac{f(0+h)-f(0)}{h} = \lim_{h\to 0}\frac{f(h)-1}{h}$$

$$\lim_{h\to 0^-}\frac{f(h)-1}{h} = \lim_{h\to 0}\frac{ah+1-1}{h} = \lim_{h\to 0}\frac{ah}{h} = a$$

$$\lim_{h\to 0^+}\frac{f(h)-1}{h} = \lim_{h\to 0^+}\frac{h^2+1-1}{h} = \lim_{h\to 0^+} h = 0$$

Need $a=0$

5.(6 pts.) Compute $\lim\limits_{x\to 0} \dfrac{\tan 2x}{\sin 3x}$.

$x \neq 0$

(a) 1/3 (b) 2 (c) 0 (d) 2/3 (e) 1

$$\frac{\tan(2x)}{\sin(3x)} = \frac{\sin(2x)}{\cos(2x)} \cdot \frac{1}{\sin(3x)} = \frac{\sin(2x)}{2x} \cdot 2x \cdot \frac{1}{\cos(2x)} \cdot \frac{3x}{\sin(3x)} \cdot \frac{1}{3x}$$

$$= \frac{\sin(2x)}{2x} \cdot \frac{1}{\cos(2x)} \cdot \frac{3x}{\sin(3x)} \cdot \frac{2\cancel{x}}{3\cancel{x}}$$

$$\Rightarrow \lim_{x\to 0} \frac{\tan(2x)}{\sin(3x)} = 1 \cdot 1 \cdot 1 \cdot \frac{2}{3} = \frac{2}{3}$$

6.(6 pts.) Compute $\lim\limits_{x\to -\infty} \dfrac{\sqrt{4x^2+x+1}}{3x-1}$.

(a) 0 (b) $-2/3$ (c) 2/3 (d) 1/3 (e) $-1/3$

NB: $\frac{1}{x} < 0$

$$\lim_{x\to -\infty} \frac{\sqrt{4x^2+x+1}}{3x-1} = \lim_{x\to -\infty} \frac{\sqrt{4x^2+x+1}\left(\frac{1}{x}\right)}{(3x-1)\left(\frac{1}{x}\right)} = \lim_{x\to -\infty} \frac{-\sqrt{4+\frac{1}{x}+\frac{1}{x^2}}}{3-1}$$

$$= \frac{-\sqrt{4}}{3} = -\frac{2}{3}$$

(*)

7.(6 pts.) Compute the tangent line to the ellipse given by the equation $x^2 + 4y^2 = 5$ at the point $(1, -1)$

(a) $y = \frac{1}{2}x - \frac{3}{2}$

(b) The tangent line does not exist.

(c) $y = \frac{1}{4}x - \frac{5}{4}$

(d) $y = \frac{1}{4}x - \frac{3}{4}$

(e) $y = -\frac{1}{4}x - \frac{3}{4}$

$(*) \Rightarrow 2x + 8yy' = 0 \xRightarrow{@(1,-1)} 2(1) + 8(-1)y' = 0 \Rightarrow \frac{dy}{dx}(1,-1) = \frac{1}{4}$

Hence L: $y - (-1) = \frac{1}{4}(x - 1)$

$\Rightarrow y = \frac{x}{4} - \frac{1}{4} - 1 = \frac{x}{4} - \frac{5}{4}$

8.(6 pts.) Let $F(x) = f(g(x))$. Compute $F'(2)$ using the following information:
$f(-1) = -3$, $f(2) = 12$, $g(-1) = -7$, $g(2) = -1$,
$f'(-1) = 2$, $f'(2) = 8$, $g'(-1) = -1$, $g'(2) = 5$.

(a) 10 (b) -15 (c) 40 (d) 2 (e) 52

$F'(x) = f'(g(x))\,g'(x)$

$\Rightarrow F'(2) = f'(g(2)) \cdot g'(2) = f'(-1) \cdot 5 = 2(5) = 10$

9.(6 pts.) For $y = (\sin 4x)^8$, compute y'.

(a) $32(\cos 4x)^7$

(b) $8(\cos 4x)^7$

(c) $8(\sin 4x)^7$

(d) $32(\sin 4x)^7$

(e) $32(\sin 4x)^7 \cos 4x$

$$y' = 8(\sin(4x))^7 \cdot 4\cos(4x) \cdot 4$$

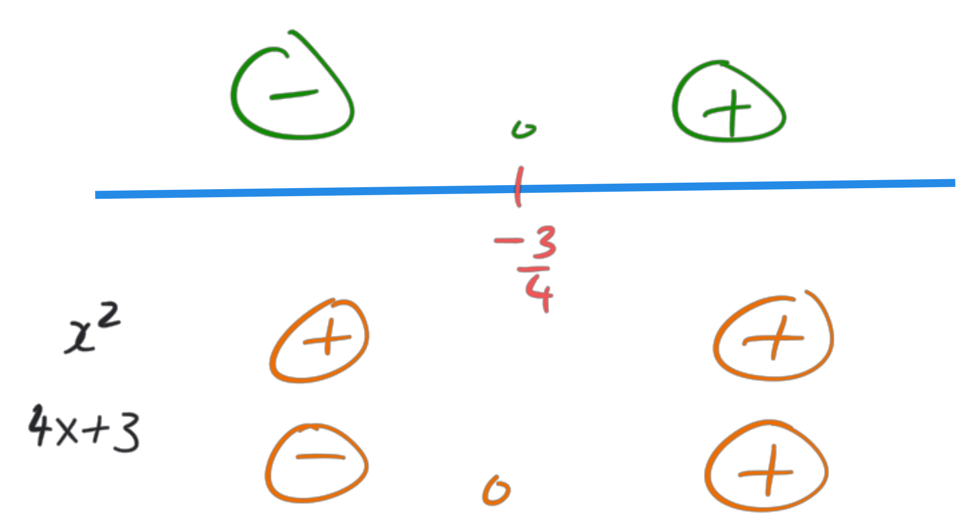
$$= 32\cos(4x)(\sin(4x))^7$$

10.(6 pts.) How many inflection points does the curve $y = \dfrac{x^5}{5} + \dfrac{x^4}{4}$ have?

(a) 1 (b) 0 (c) 3 (d) 2 (e) 4

$$y' = x^4 + x^3$$

$$y'' = 4x^3 + 3x^2 = x^2(4x+3)$$



inflection pt @ $x = \frac{-3}{4}$.

11.(6 pts.) Compute the derivative y' for the curve $\sqrt{x^2+y^2} = 2+y$ at the point $x=4$, $y=3$.

(a) $2/11$ (b) -2 (c) 2 (d) 0 (e) $-2/11$

$$\frac{1}{2}(x^2+y^2)^{-\frac{1}{2}}(2x+2yy') = y'$$

$x=4$
$y=3$

$$\frac{1}{2}(25)^{-1/2}(8+6y') = y'$$

$$8+6y' = 10y'$$

$$8 = 4y'$$

$$2 = y'$$

12.(6 pts.) A kite 100 ft above the ground is flying horizontally (away from its holder) with a speed of 16ft/sec. At what rate is the angle between the string and the horizontal direction changing, when 200 ft of the string have been let out?

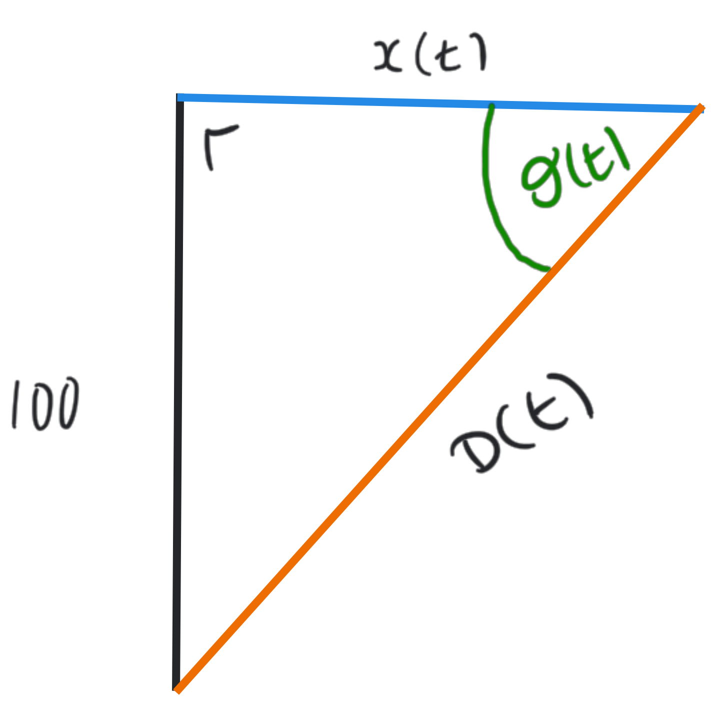
(a) $\frac{\pi}{50}$ radian/second (b) $\frac{1}{25}$ radian/second

(c) $\frac{1}{50}$ radian/second (d) $-\frac{1}{25}$ radian/second

(e) $-\frac{1}{50}$ radian/second

$\dot{x}(t) = 16$ ft/sec.

$D(t_*) = 200$ ft $\Rightarrow$ $x(t_*)^2 = (200)^2 - 100^2 = 30,000$



$$\tan(\theta(t)) = \frac{100}{x(t)}$$

$$\Rightarrow \sec^2(\theta(t_*))\cdot\dot{\theta}(t_*) = \frac{-100}{x(t_*)^2}\cdot\dot{x}(t_*)$$

$$\left(\frac{D(t_*)}{x(t_*)}\right)^2\cdot\dot{\theta}(t_*) = \frac{-100}{x(t_*)^2}\cdot\dot{x}(t_*)$$

$$\Rightarrow \left(\frac{4}{3}\right)\cdot\dot{\theta}(t_*) = \frac{-1}{300}\cdot 16 \Rightarrow \dot{\theta}(t_*) = \frac{-1}{25} \text{ rad/s}$$

13.(6 pts.) Find the linearization of $f(x) = \sqrt{10 - x^2}$ at $a = -1$.

(a) $L(x) = \frac{2}{3}(x+1) + 3$

(b) $L(x) = -\frac{2}{3}(x+1) + 3$

(c) $L(x) = x + 4$

(d) $L(x) = -\frac{1}{3}(x+1) + 3$

(e) $L(x) = \frac{1}{3}(x+1) + 3$

$h_a(x) = f'(a)(x-a) + f(a)$:

$f'(x) = \frac{1}{2}(10-x^2)^{-1/2} \cdot (-2x) = \frac{-x}{\sqrt{10-x^2}}$

$f'(-1) = \frac{1}{\sqrt{9}} = \frac{1}{3}$

$f(-1) = \sqrt{10-(-1)^2} = \sqrt{9} = 3$

$h(x) = \frac{1}{3}(x-(-1)) + 3 = \frac{1}{3}(x+1) + 3$

14.(6 pts.) Find all local maxima and minima of the function $f(x) = 2|x| - x^2 - 1$.

(a) Local maxima: $(x, y) = (-1, 0)$ and $(x, y) = (1, 0)$, local minimum $(x, y) = (0, -1)$.

(b) Only local minimum at $(x, y) = (0, -1)$, no local maxima.

(c) Local maximum: $(x, y) = (-1, 0)$, local minimum $(x, y) = (0, -1)$.

(d) No local maxima or minima, because the function $|x|$ has no derivative at $x = 0$.

(e) Local maxima: $(x, y) = (-1, 0)$ and $(x, y) = (1, 0)$, no local minimum.

$$f(x) = 2|x| - x^2 - 1 = \begin{cases} 2x - x^2 - 1 & \text{for } x \geq 0 \\ -2x - x^2 - 1 & \text{for } x < 0 \end{cases}$$

$$f'(x) = \begin{cases} 2 - 2x & \text{for } x > 0 \\ -2 - 2x & \text{for } x < 0 \end{cases}$$

← not defined at $x = 0$

$$f''(x) = \begin{cases} -2 & \text{for } x > 0 \\ -2 & \text{for } x < 0 \end{cases}$$

$\Rightarrow$ any critical pts other than @ 0 are local max.s.

15.(6 pts.) Find all asymptotes of the curve $y = \dfrac{2x^2 + x + 1}{x - 1}$.

(a) vertical asymptote $x = 1$, no other asymptotes.

(b) slant asymptote $y = 2x + 1$, vertical asymptote $x = 1$, no horizontal asymptotes.

(c) horizontal asymptotes $y = 2$, slant asymptote $y = 2x + 3$, no vertical asymptotes.

(d) slant asymptote $y = 2x + 3$, vertical asymptote $x = 1$, no horizontal asymptotes.

(e) horizontal asymptotes $y = 2$, vertical asymptote $x = 1$, no slant asymptotes.

$2(1)^2 + 1 + 1 = 4 \Rightarrow$ vertical asymptote @ $x = 1$

$$\begin{array}{r l}
 & 2x + 3 + \frac{4}{x-1} \\
x - 1 \,\big) & 2x^2 + x + 1 \\
 & -(2x^2 - 2x) \\
\hline
 & 3x + 1 \\
 & -(3x - 3) \\
\hline
 & 4
\end{array}$$

$\Rightarrow$ slant asymptote : $y = 2x + 3$

16.(6 pts.) Find **all** the points on the hyperbola $y^2 - x^2 = 4$ that are closest to the point $(2, 0)$.

(a) $(1, \pm 5)$

(b) $(1, \pm\sqrt{5})$

(c) $(-1, \sqrt{5})$

(d) $(1, \sqrt{5})$

(e) $(\sqrt{5}, 1)$

$D^2 = (x-2)^2 + y^2 = (x-2)^2 + x^2 + 4 = 2x^2 - 4x + 8$

$\dfrac{d(D^2)}{dx} = 4x - 4 \Rightarrow$ critical pts for $x = 1$

$\dfrac{d^2(D^2)}{dx^2} = 4$

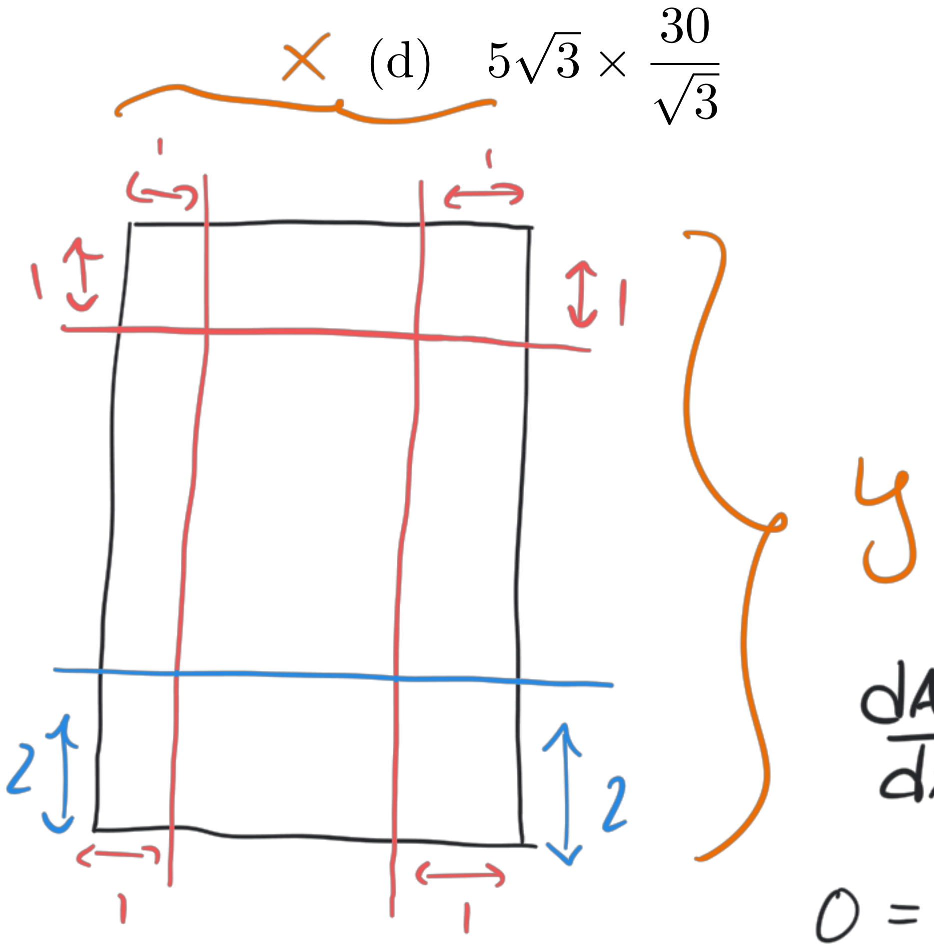
Min

$x = 1 \Rightarrow y^2 = (1)^2 + 4 \rightarrow y = \pm\sqrt{5}$

17.(6 pts.) A page of a book is to have a total area of 150 square inches, with 1 inch margins at the top and sides, and a 2 inch margin at the bottom. Find the dimensions in inches of the page which will have the largest print area.

(a) $3\sqrt{7} \times \frac{50}{\sqrt{7}}$ (b) 5×30 (c) $11\frac{7}{13} \times 13$

(d) $5\sqrt{3} \times \frac{30}{\sqrt{3}}$ (e) 10×15



$$xy = 150 \Rightarrow y = \frac{150}{x}$$

Maximise: $(x-2)(y-3) = A_p$

$$A_p = (x-2)\left(\frac{150}{x} - 3\right) = 150 - 3x - \frac{300}{x} + 6$$

$$\frac{dA_p}{dx} = -3 + \frac{300}{x^2} \quad : \quad \boxed{\frac{d^2A_p}{dx^2} = -\frac{300}{x^3} < 0 \text{ for } x>0 \Rightarrow \text{Max}}$$

$$0 = -3 + \frac{300}{x^2} \Rightarrow x = 10 \Rightarrow y = 15$$

18.(6 pts.) Newton's method is to be used to find a root of the equation

$$x^3 - x - 1 = 0.$$

If $x_1 = 1$, find x_2.

(a) 1.50 (b) 0.95 (c) 3 (d) 1.35 (e) 1.75

$$f(x) = x^3 - x - 1 \Rightarrow f'(x) = 3x^2 - 1$$

$$f(1) = -1, \quad f'(1) = 2$$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} = 1 - \frac{-1}{2} = \frac{3}{2}$$

19.(6 pts.) Express the limit below as a definite integral.

$$\lim_{n\to\infty}\sum_{i=1}^{n}\frac{\pi}{4n}\sec^2\left(\frac{i\pi}{4n}\right) = \lim_{n\to\infty}\sum_{i=1}^{n}\frac{(\pi/4)}{n}\sec^2\left(\frac{(\pi/4)}{n}i\right)$$

(a) $\int_0^1 \sec^2\left(\frac{\pi}{4}x\right)dx$

(b) $\frac{\pi}{4}\int_0^{\pi/4}\sec^2(x)dx$

(c) $\int_0^{\pi/4}\sec^2\left(\frac{\pi}{4}\right)dx$

(d) $\int_0^{\pi/2}\sec^2(x)dx$

(e) $\int_0^{\pi/4}\sec^2(x)dx$

$$\Delta x = \frac{\pi/4}{n} = \frac{\pi/4 - 0}{n}$$

$$f(x) = \sec^2(x)$$

$$\sum \Delta x\, f(x_i)$$
$$= \sum \Delta x\, f(i\Delta x)$$

$$\int_0^{\pi/4}\sec^2(x)\,dx$$

20.(6 pts.) If $f(x) = \int_0^{5x}\cos(u^2)du$, find $f'(x)$.

(a) $-\cos(5x^2)$

(b) $5\cos(25x^2)$

(c) $-25\cos(5x^2)$

(d) $5\cos(5x^2)$

(e) $-5\cos(25x^2)$

$$f'(x) = \cos(25x^2)\cdot 5$$

21.(6 pts.) Evaluate the integral $\int_0^{\sqrt{\pi}} x\sin(x^2)dx$.

(a) $\frac{\pi}{4}$ (b) 2 (c) $\frac{1}{4}$ (d) $1-\frac{1}{\pi}$ (e) 1

$u = x^2$

$\frac{du}{dx} = 2x$

$\Rightarrow \frac{1}{2}du = xdx$

$x=0 \Rightarrow u=0$

$x=\sqrt{\pi} \Rightarrow u=\pi$

$$\int_0^{\sqrt{\pi}} x\sin(x^2)dx = \int_0^{\pi} \frac{1}{2}\sin(u)\,du = -\frac{1}{2}\cos(u)\Big|_0^{\pi}$$

$$= -\frac{1}{2}\cos(\pi) + \frac{1}{2}\cos(0) = \frac{1}{2}+\frac{1}{2} = 1$$

22.(6 pts.) Which of the following integrals give the area of the region below the curve $y = 2x$ and above the curve $y = x^2 - 4x$?

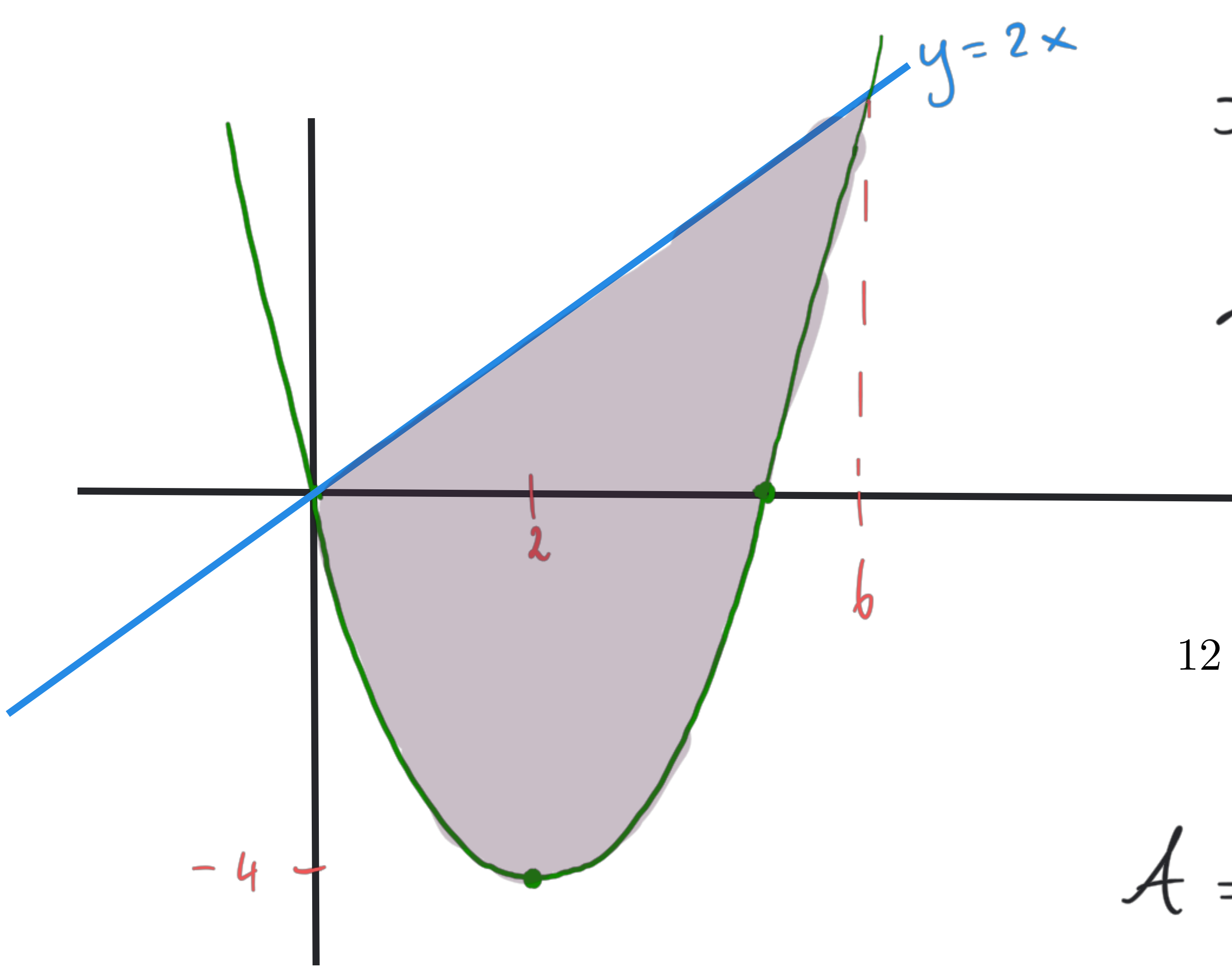
(a) $\int_0^4 \Big((x^2-4x)-2x\Big)dx$

(b) $\int_0^6 \Big((x^2-4x)-2x\Big)dx$

(c) $\int_0^6 \Big(2x-(x^2-4x)\Big)dx$

(d) $\int_0^4 \Big(2x-(x^2-4x)\Big)dx$

(e) $\int_0^4 \Big(2x-(x^2-4x)\Big)dx + \int_4^6 \Big((x^2-4x)-2x\Big)dx$



$x^2 - 4x = (x-2)^2 - 4$

$2x = x^2 - 4x$

$\Rightarrow x^2 - 6x = 0$

$\Rightarrow x(x-6) = 0$

$\Rightarrow x = 0$ or $x = 6$

$$A = \int_0^6 \big(2x - (x^2-4x)\big)dx$$

23.(6 pts.) An area in xy plane bounded by the curves $y = 0$ and $y = x - x^2$. If we rotate this area about $x = 7$, which integral below gives the volume?

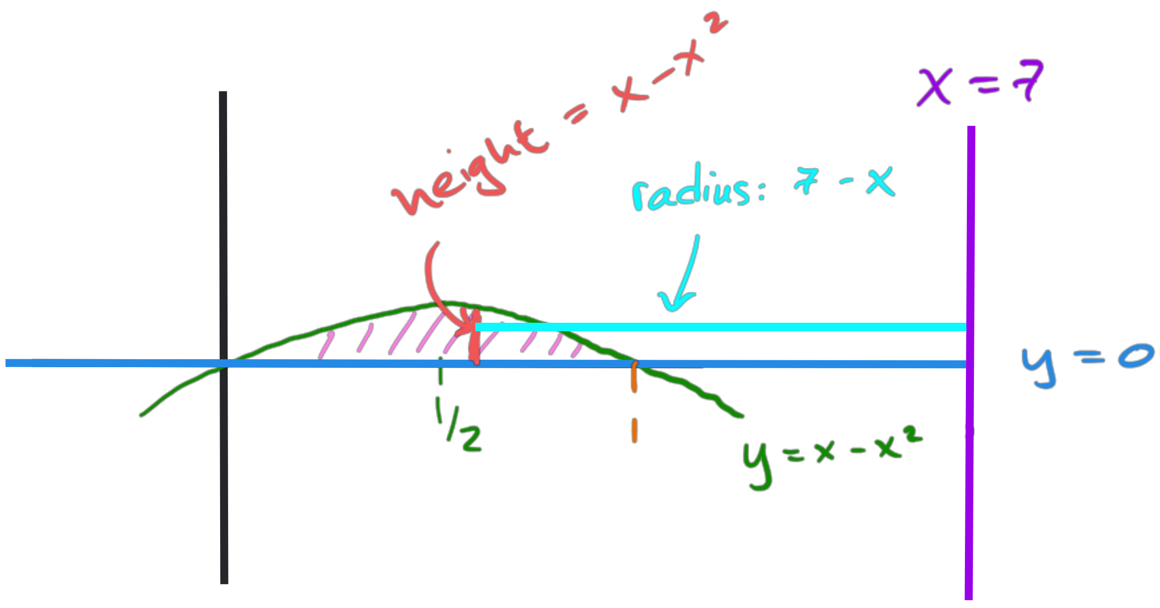
(a) $\pi \int_0^{1/4} (x - x^2)^2 \, dx$

(b) $2\pi \int_0^1 (7 - x)(x - x^2) \, dx$

(c) $2\pi \int_0^{\pi} (x - x^2 - 7) \, dx$

(d) $\pi \int_0^1 (x - x^2)^2 \, dx$

(e) $2\pi \int_0^1 (x - 7)(x - x^2) dx$



$$x - x^2 = -(x - \tfrac{1}{2})^2 + \tfrac{1}{4}$$

$$x - x^2 = 0 \Rightarrow x = 0 \text{ or } x = 1$$

$$V = \int_0^1 2\pi(7 - x)(x - x^2) dx$$

24.(6 pts.) The plane region bounded by the curves $y = 2$ and $y = 2 + 2x - x^2$ is rotated about the x axis. Which integral below gives the volume?

(a) $\pi \int_0^2 \left(4 - (2 + 2x - x^2)^2\right) dx$

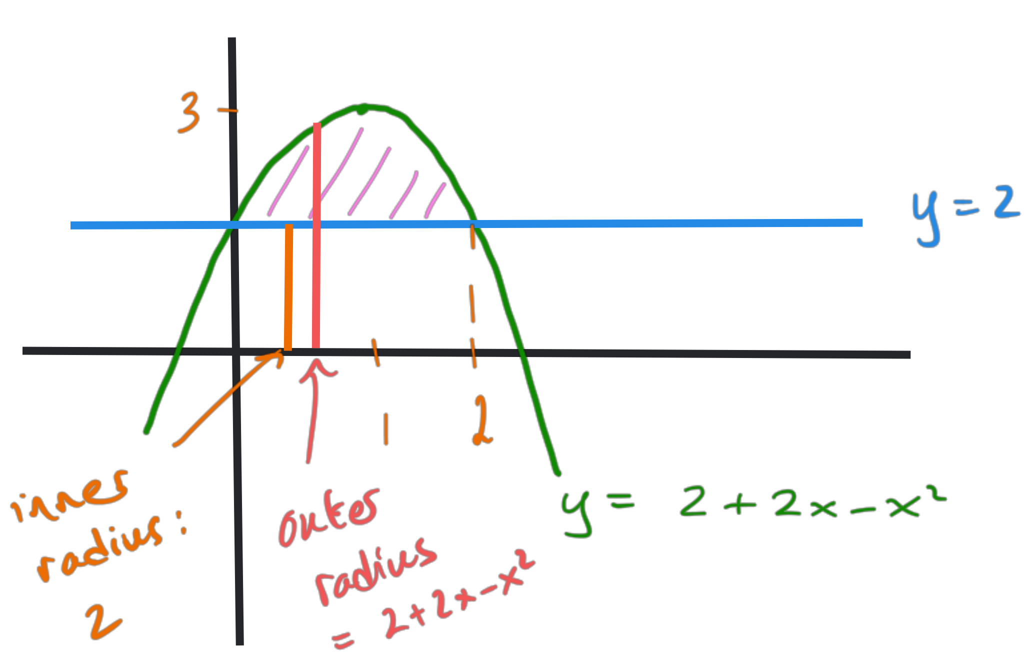
(b) $\pi \int_0^2 \left((2 + 2x - x^2)^2 - 4\right) dx$

(c) $2\pi \int_0^2 \left((2 + 2x - x^2) - 2\right) dx$

(d) $\pi \int_0^1 \left((2 + 2x - x^2)^2 - 4\right) dx$

(e) $\pi \int_0^1 \left(4 - (2 + 2x - x^2)^2\right) dx$

$$y = 2 + 2x - x^2 = -(x - 1)^2 + 3$$



$$2 + 2x - x^2 = 2$$

$$\Rightarrow 2x - x^2 = 0$$

$$\Rightarrow (2 - x)(x) = 0$$

$$\Rightarrow x = 0 \text{ or } x = 2$$

$$V = \int_0^2 \pi(2 + 2x - x^2)^2 dx - \int_0^2 \pi(2)^2 dx = \pi \int_0^2 \left[(2 + 2x - x^2)^2 - 4\right] dx$$

25.(6 pts.) The function $f(x) = \sqrt{16-2x}$ is continuous on the interval $[0, 8]$. Which number below is its average value on this interval?

(a) $\frac{8}{3}$ (b) $\frac{64}{3}$ (c) $\frac{8}{3}\sqrt{8}$

(d) $\frac{16}{3}$ (e) $-\frac{8}{3}$

$$f_{ave} = \frac{1}{8-0}\int_0^8 \sqrt{16-2x}\,dx = \frac{1}{8}\int_0^8 \sqrt{16-2x}\,dx$$

$u = 16-2x$

$\Rightarrow \frac{du}{dx} = -2$

$\Rightarrow -\frac{1}{2}du = dx$

$x=0 \Rightarrow u=16$

$x=8 \Rightarrow u=0$

$$= \frac{1}{8}\int_{16}^{0} \sqrt{u}\left(\frac{-1}{2}\right)du$$

$$= \frac{-1}{16}\left(\frac{u^{3/2}}{3/2}\Big|_{16}^{0}\right)$$

$$= \frac{-1}{16}\left(0 - \left(\frac{2}{3}\cdot 64\right)\right)$$

$$= \frac{8}{3}$$

Name: ____________________

Instructor: ANSWER

Math 10550, Practice Final Exam, December

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 2 hours.
- Be sure that your name is on this page.
- Be sure that you have all 25 problems.
- This is the only page you need to hand in.

Please mark your answers with an **X**! Do NOT circle them!

1.	•	b	c	d	e	15.	a	b	c	•	e
2.	a	b	c	d	•	16.	a	•	c	d	e
3.	a	b	c	d	•	17.	a	b	c	d	•
4.	a	•	c	d	e	18.	•	b	c	d	e
5.	a	b	c	•	e	19.	a	b	c	d	•
6.	a	•	c	d	e	20.	a	•	c	d	e
7.	a	b	•	d	e	21.	a	b	c	d	•
8.	•	b	c	d	e	22.	a	b	•	d	e
9.	a	b	c	d	•	23.	a	•	c	d	e
10.	•	b	c	d	e	24.	a	•	c	d	e
11.	a	b	•	d	e	25.	•	b	c	d	e
12.	a	b	c	•	e						
13.	a	b	c	d	•						
14.	•	b	c	d	e						

Final Exam: ____________

Previous Total: ____________

Course Total: ____________