

Name: _____

**Math 10550, Final Exam Fall 2014:
December 18, 2016**

Instructor: _____

- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.
- No calculators are to be used.
- The exam lasts for two hours.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 14 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- | | |
|-------------------------|-------------------------|
| 1. (a) (b) (c) (d) (e) | 15. (a) (b) (c) (d) (e) |
| 2. (a) (b) (c) (d) (e) | 16. (a) (b) (c) (d) (e) |
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| 9. (a) (b) (c) (d) (e) | 23. (a) (b) (c) (d) (e) |
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| 13. (a) (b) (c) (d) (e) | |
| 14. (a) (b) (c) (d) (e) | |

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Multiple Choice

1.(6 pts.) Find the limit

$$\lim_{x \rightarrow 3^-} \frac{x^2 - 4x + 3}{x - 3}$$

- (a) 1 (b) ∞ (c) $-\infty$ (d) 2 (e) -1

$$\lim_{x \rightarrow 3^-} \frac{x^2 - 4x + 3}{x - 3} = \lim_{x \rightarrow 3^-} \frac{(x-1)(\cancel{x-3})}{(\cancel{x-3})} = \lim_{x \rightarrow 3^-} (x-1) = 2$$

2.(6 pts.) Find the limit

$$\lim_{x \rightarrow 2^+} \frac{\sqrt{x^2 - 1} - \sqrt{x + 1}}{x - 2}$$

- (a) $-\infty$ (b) ∞ (c) $\frac{1}{2\sqrt{3}}$ (d) 3 (e) $\frac{\sqrt{3}}{2}$

$$\lim_{x \rightarrow 2^+} \frac{\sqrt{x^2 - 1} - \sqrt{x + 1}}{x - 2} = \lim_{x \rightarrow 2^+} \frac{\sqrt{(x-1)(x+1)} - \sqrt{x+1}}{x - 2}$$

$$= \lim_{x \rightarrow 2^+} \frac{(\sqrt{x-1} - 1)\sqrt{x+1}}{x - 2} = \lim_{x \rightarrow 2^+} \frac{\sqrt{x+1}(\sqrt{x-1} - 1)(\sqrt{x-1} + 1)}{(x-2)(\sqrt{x-1} + 1)}$$

$$= \lim_{x \rightarrow 2^+} \frac{\sqrt{x+1}(\cancel{x-1} - 1)2}{(\cancel{x-2})(\sqrt{x-1} + 1)} = \frac{\sqrt{3}}{2}$$

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3.(6 pts.) Find the equation of the tangent line to the curve $y = \sqrt{2x+1}$ at $x = 4$.

(a) $y = \frac{2}{3}x + \frac{1}{3}$

(b) $y = \frac{2}{3}x - \frac{8}{3}$

(c) $y = \frac{1}{3}x + 4$

(d) $y = 2x + 7$

(e) $y = \frac{1}{3}x + \frac{5}{3}$

$x = 4 \Rightarrow y = \sqrt{9} = 3$ pt: $(4, 3)$

$\frac{dy}{dx} = \frac{1}{2}(2x+1)^{-1/2} \cdot 2 = \frac{1}{\sqrt{2x+1}} : \frac{dy}{dx} \Big|_{x=4} = \frac{1}{3}$

L: $y - 3 = \frac{1}{3}(x - 4) \Rightarrow y = \frac{x}{3} + \frac{5}{3}$

4.(6 pts.) For what value of a is the function f given by

$$f(x) = \begin{cases} \frac{2-x}{x^2-3x+2} & x \neq 2 \\ a & x = 2 \end{cases}$$

at $x=2$

continuous ~~everywhere~~?

(a) $a = 1$

(b) $a = -1$

(c) $a = 2$

(d) $a = 0$

(e) No value of a makes f continuous everywhere

no value of a
makes f continuous
at $x=1$

$$\lim_{x \rightarrow 2} f(x) = \lim_{x \rightarrow 2} \frac{2-x}{x^2-3x+2} = \lim_{x \rightarrow 2} \frac{2-x}{(x-2)(x-1)} = \lim_{x \rightarrow 2} \frac{-1}{x-1} = -1$$

Want: $\lim_{x \rightarrow 2} f(x) = f(2)$

$\Leftrightarrow -1 = a$ 3

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5.(6 pts.) Find $f'(x)$ where

$$f(x) = \frac{\cos x}{(2x - 3)^2}$$

(a) $\frac{(\sin x)(2x - 3)^2 + 4(\cos x)(2x - 3)}{(2x - 3)^4}$

(b) $\frac{-(\sin x)}{4(2x - 3)}$

(c) $\frac{-(\sin x)(2x - 3)^2 - 4(\cos x)(2x - 3)}{(2x - 3)^4}$

(d) $\frac{-(\sin x)(2x - 3)^2 - 4(\cos x)(2x - 3)}{(2x - 3)^2}$

(e) $\frac{4(\sin x)(2x - 3) - (\sin x)(2x - 3)^2}{(2x - 3)^4}$

$$u = \cos(x) \Rightarrow \frac{du}{dx} = -\sin(x)$$

$$v = (2x - 3)^2 \Rightarrow \frac{dv}{dx} = 2(2x - 3) \cdot 2 = 4(2x - 3)$$

$$f'(x) = \frac{v \frac{du}{dx} - u \frac{dv}{dx}}{u^2}$$

6.(6 pts.) Find the equation of the line tangent to the graph of $x^4y^2 + y^3 = 2$ at the point $(1, 1)$.

(a) $y = -\frac{4}{5}x + \frac{9}{5}$

(b) $y = -\frac{1}{2}x + \frac{3}{2}$

(c) $y = \frac{4}{5}x + \frac{1}{5}$

(d) $y = -\frac{4}{5}x + \frac{4}{5}$

(e) $y = \frac{1}{2}x + \frac{1}{2}$

$$4x^3y^2 + 2x^4yy' + 3y^2y' = 0 \Rightarrow y' = \frac{-4x^3}{2x^4y + 3y^2}$$

$$y'(1, 1) = \frac{-4}{2+3} = -\frac{4}{5}$$

h: $y - 1 = -\frac{4}{5}(x - 1) \Rightarrow y = -\frac{4}{5}x + \frac{9}{5}$

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7.(6 pts.) If

$$f(x) = \sqrt{1 + \sqrt{x}}$$

what is $f'(x)$?

(a) $\frac{\sqrt{x}}{4\sqrt{1 + \sqrt{x}}}$

(b) $\frac{1}{2\sqrt{1 + \sqrt{x}}}$

(c) $\frac{\sqrt{x}}{\sqrt{1 + \sqrt{x}}}$

(d) $\frac{1}{4\sqrt{x}\sqrt{1 + \sqrt{x}}}$

(e) $\frac{1}{\sqrt{1 + \sqrt{x}}}$

$$f'(x) = \frac{1}{2}(1 + \sqrt{x})^{-1/2} \cdot \frac{1}{2}(x)^{-1/2} = \frac{1}{4\sqrt{x}\sqrt{1 + \sqrt{x}}}$$

8.(6 pts.) Find the linearization of the function $f(x) = \sqrt[5]{x}$ at $a = 32$ and use it to approximate the number $\sqrt[5]{34}$. Which of the following gives the resulting approximation?

(a) $\frac{21}{20}$

(b) $\frac{79}{40}$

(c) $\frac{19}{20}$

(d) $\frac{81}{40}$

(e) 2

$$L_a(x) = L_{32}(x) = f'(32)(x - 32) + f(32)$$

$$f(32) = \sqrt[5]{32} = 2$$

$$f'(x) = \frac{1}{5}x^{-4/5} \Rightarrow f'(32) = \frac{1}{5(\sqrt[5]{32})^4} = \frac{1}{80}$$

$$\Rightarrow L_{32}(34) = \frac{1}{80}(34 - 32) + 2 = \frac{1}{40} + 2 = \frac{81}{40}$$

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9.(6 pts.) Two cyclists are approaching a town, one cycling due east at 10 miles per hour and the other cycling due south at 15 miles per hour. How fast is the distance between the bicycles decreasing when the eastbound cyclist is 40 miles from the town and the southbound cyclist is 30 miles from the town?

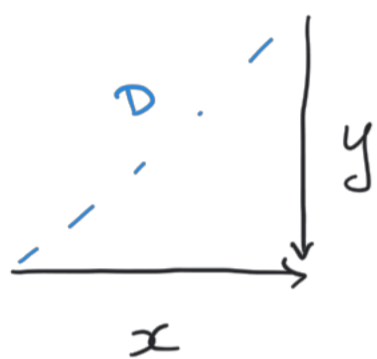
(a) 17 m. p.h.

(b) 32 m.p.h.

(c) 15 m.p.h.

(d) 20 m.p.h.

(e) 12 m.p.h.



$$\begin{aligned} \dot{x} &= -10 \text{ mph} & | & \dot{D} = \frac{1}{2} (x^2 + y^2)^{-1/2} (\cancel{2x}\dot{x} + \cancel{2y}\dot{y}) \\ \dot{y} &= -15 \text{ mph} & | & \\ D &= \sqrt{x^2 + y^2} & | & \dot{D}_* = (1600 + 900)^{-1/2} (40(-10) + 30(-15)) \\ & & | & = -\frac{850}{50} = -17 \end{aligned}$$

10.(6 pts.) Among all positive x values (for $0 < x < \infty$), find the value of x which minimizes

$$y = f(x) = x + \frac{3}{x}$$

(a) $x = 1.5$

(b) $x = \sqrt{2}$

(c) $x = 1$

(d) $x = \sqrt{3}$

(e) No such value of x exists.

$$\frac{dy}{dx} = 1 - \frac{3}{x^2} \leftarrow \text{critical point at } x = 0, \text{ but } x > 0.$$

$$\frac{dy}{dx} = 0 \quad @ \quad x = \pm \sqrt{3}$$

$$\frac{d^2y}{dx^2} = \frac{6}{x^3} > 0 \text{ for all } x > 0 \text{ Hence, minimums.}$$

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But only $\sqrt{3} > 0$.

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11.(6 pts.) Let

$$f(x) = \frac{|x|}{x^2 + 1}.$$

Which of the following statements is true about $f(x)$?

- (a) $f(x)$ has a local maximum at $x = 1$, and no local minima.
- (b) There is a local minimum at $x = -1$ and a local maximum at $x = 1$
- (c) $f(x)$ has no critical points besides $x = -1$ and $x = 1$.
- (d) $f(x)$ has no critical points.
- (e) There are local maxima at $x = \pm 1$, and a local minimum at $x = 0$

$$f'(x) = \begin{cases} \frac{(x^2+1)(1) - (x)(2x)}{(x^2+1)^2} & , x > 0 \\ \frac{(x^2+1)(-1) - (-x)(2x)}{(x^2+1)^2} & , x < 0 \end{cases}$$
$$= \begin{cases} \frac{1-x^2}{1+x^2} & , x > 0 \\ \frac{x^2-1}{1+x^2} & , x < 0 \end{cases}$$

Critical pts : $x = \pm 1$

A sign chart for the derivative f'(x) is shown. The x-axis has tick marks at -1, 0, and 1. Above the axis, there are four circles containing signs: a plus sign (+) at x < -1, a minus sign (-) between x = -1 and x = 0, a plus sign (+) between x = 0 and x = 1, and a minus sign (-) at x > 1. A red arrow points upwards from the x-axis at x = 0, with the handwritten note: "f'(0) not defined, but still local min".

12.(6 pts.) How many inflection points does the graph of $f(x) = \sin(x) - \cos(x)$ have on the interval $[0, \pi]$.

- (a) 2
- (b) 1
- (c) 3
- (d) 4
- (e) 0

$$0 = f''(x) = -\sin(x) + \cos(x)$$

$$\Rightarrow \sin(x) = \cos(x)$$

$$\Rightarrow x = \frac{\pi}{4}$$

$$f''(0) = 1$$

$$f''(\pi) = -1$$

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13.(6 pts.) Find the limit

$$\lim_{x \rightarrow 0^+} \frac{\sin(x^2)}{x} = \lim_{x \rightarrow 0^+} \frac{\sin(x^2)}{x^2} \cdot x = (1) \cdot 0$$

(a) 0

(b) $-\infty$

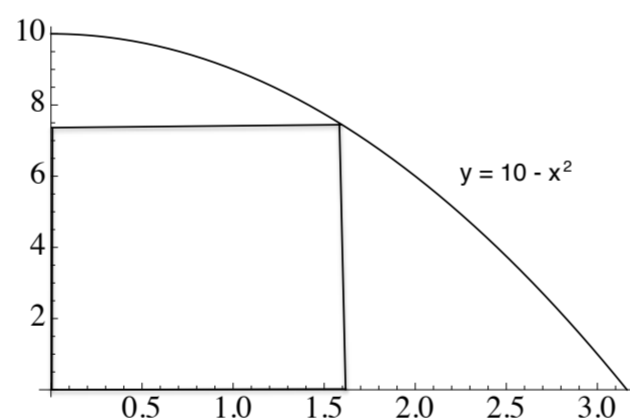
(c) 1

(d) $+\infty$

(e) Does not exist and is not equal to $\pm\infty$

= 0

14.(6 pts.) Consider the picture shown below. You wish to fit a rectangle of maximal area into the region bounded by the curve $y = 10 - x^2$ and the lines $y = 0$ and $x = 0$, such that one corner of the rectangle is positioned at the point $(0, 0)$ and the opposite corner touches the graph of the curve as shown. What are the dimensions of such a rectangle of maximal area?



(a) $\frac{\sqrt{10}}{2} \times \frac{30}{4}$

(b) 2×6

(c) $\sqrt{\frac{10}{3}} \times \frac{20}{3}$

(d) $\sqrt{3} \times 7$

(e) $\frac{\sqrt{10}}{2} \times \frac{\sqrt{10}}{2}$

$$A = x(10 - x^2) = 10x - x^3 \Rightarrow \frac{dA}{dx} = 10 - 3x^2$$

Critical pt @ $x = \sqrt{\frac{10}{3}}$

8

$$y = 10 - \frac{10}{3} = \frac{20}{3}$$

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15.(6 pts.) The equation $\tan(x) - \sin(2x) - \frac{1}{2} = 0$ has one solution between 0 and 1.

Find the result of one iteration of Newton's Method applied to this equation with 0 as the starting point. (i.e. find x_2 using Newton's method applied to the equation with $x_1 = 0$).

- (a) $\frac{1}{2}$ (b) $-\frac{1}{3}$ (c) -1 (d) $\frac{1}{3}$ (e) $-\frac{1}{2}$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)} \quad ; \quad x_2 = 0 - \frac{f(0)}{f'(0)} = - \frac{(\tan(0) - \sin(0) - \frac{1}{2})}{\sec^2(0) - 2\cos(0)}$$
$$= - \frac{(-\frac{1}{2})}{1 - 2} = -\frac{1}{2}$$

16.(6 pts.) Consider the following rational function:

$$f(x) = \frac{(x-1)(x^2+1)}{x^2+x-2} = \frac{(x-1)(x^2+1)}{(x-1)(x+2)}$$

Which of the statements shown below is true?

- (a) $x = 1$ is a vertical asymptote of f .
(b) $y = x - 2$ is a slant asymptote of f .
(c) $y = 1$ is a horizontal asymptote of f .
(d) $y = 2$ is a horizontal asymptote of f .
(e) $y = x + 2$ is a slant asymptote of f .

$$x+2 \overline{) \begin{array}{r} x^2 + 1 \\ -x^2 + 2x \\ \hline -2x + 1 \\ +2x - 4 \\ \hline 5 \end{array}}$$

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17.(6 pts.) Consider the definite integral

$$\int_1^3 x^2 + 1 dx.$$

Which of the following Riemann sums gives the **right end point approximation** to the above integral, using four approximating rectangles?

(a) $\frac{1}{2} \left(2 + \frac{13}{4} + 5 + \frac{29}{4} \right)$

(b) $\frac{1}{2} \left(\frac{9}{4} + 4 + \frac{25}{4} + 9 \right)$

(c) $\frac{1}{2} \left(\frac{13}{4} + 5 + \frac{29}{4} + 10 \right)$

(d) $\frac{1}{2} \left(1 + \frac{9}{4} + 4 + \frac{25}{4} \right)$

(e) $\frac{1}{2} \left(\frac{25}{4} + 9 + \frac{49}{4} + 16 \right)$

$$\int_1^3 x^2 + 1 dx \approx R_4 = \frac{2}{4} \left(f\left(\frac{3}{2}\right) + f(2) + f\left(\frac{5}{2}\right) + f(3) \right)$$
$$= \frac{1}{2} \left(\frac{13}{4} + 5 + \frac{29}{4} + 10 \right)$$

18.(6 pts.) Let

$$F(x) = \int_1^{x^3} \sqrt{1 + \sin^2(u)} du.$$

Which of the following gives $F'(x)$?

(a) $\frac{\sin(x^3) \cos(x^3)}{\sqrt{1 + \sin^2(x^3)}}$

(b) $\frac{\sin(x) \cos(x)}{\sqrt{1 + \sin^2(x)}}$

(c) $3x^2 \sqrt{1 + \sin^2(x)}$

(d) $\frac{3x^2 \sin(x^3) \cos(x^3)}{\sqrt{1 + \sin^2(x^3)}}$

(e) $3x^2 \sqrt{1 + \sin^2(x^3)}$

$$F'(x) = \sqrt{1 + \sin^2(x^3)} \cdot 3x^2$$

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19.(6 pts.) The function $F(x)$ has the following properties:

$$F'(x) = \sin(4x) \quad \text{and} \quad F(0) = 3/4.$$

Find $F\left(\frac{\pi}{8}\right)$.

- (a) $\frac{7}{4}$ (b) 0 (c) 1 (d) $\frac{3}{4}$ (e) 2

$$F(x) = -\frac{\cos(4x)}{4} + C \quad \Bigg| \Rightarrow F\left(\frac{\pi}{8}\right) = -\frac{\cos\left(\frac{\pi}{2}\right)}{4} + 1 = 1$$

$$\frac{3}{4} = F(0) = -\frac{\cos(0)}{4} + C \Rightarrow C = 1$$

20.(6 pts.) Evaluate the following definite integral

$$\int_0^1 \frac{x^2}{\sqrt{2x^3+1}} dx$$

- (a) $\frac{\sqrt{3}-1}{3}$ (b) $2\sqrt{3}-1$ (c) $\frac{1}{3}$ (d) $\frac{\sqrt{2}-1}{3}$ (e) $\frac{1}{6}$

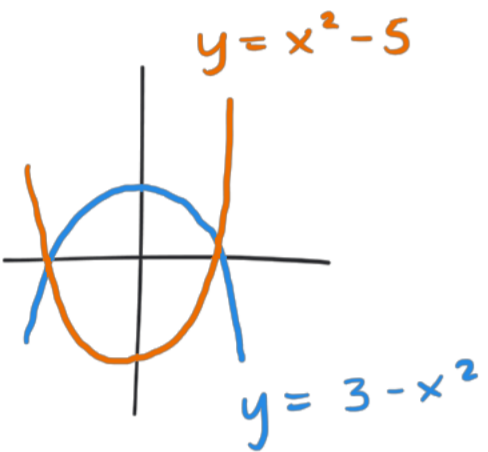
$$\begin{aligned} u &= 2x^3 + 1 \\ \frac{du}{dx} &= 6x^2 \\ \Rightarrow \frac{1}{6} du &= x^2 dx \\ x=0 &\Rightarrow u=1 \\ x=1 &\Rightarrow u=3 \end{aligned} \quad \left. \begin{aligned} \int_0^1 \frac{x^2}{\sqrt{2x^3+1}} dx &= \frac{1}{6} \int_1^3 u^{-1/2} du \\ &= \frac{1}{6} \cdot 2\sqrt{u} \Big|_1^3 \\ &= \frac{1}{6} (2\sqrt{3} - 2) \\ &= \frac{\sqrt{3}-1}{3} \end{aligned} \right\}$$

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21.(6 pts.) Find the area between the curves $y = 3 - x^2$ and $y = x^2 - 5$.

- (a) $\frac{64}{3}$ (b) 0 (c) 16 (d) 32 (e) $\frac{32}{3}$



pts of intersection: $3 - x^2 = x^2 - 5$

$$\Rightarrow 2x^2 = 8 \Rightarrow x = \pm 2$$

$$\text{Area} = \int_{-2}^2 (3 - x^2) - (x^2 - 5) dx = \int_{-2}^2 8 - 2x^2 dx$$

$$= 8x - \frac{2x^3}{3} \Big|_{-2}^2 = \left(16 - \frac{16}{3}\right) - \left(-16 + \frac{16}{3}\right) = 2\left(\frac{32}{3}\right) = \frac{64}{3}$$

22.(6 pts.) A region in the xy plane is bounded by the curves $y = \cos x$, $y = \sin x$, $x = 0$ and $x = \frac{\pi}{4}$. Which integral below gives the volume of the solid obtained by rotating the given region about the x -axis?

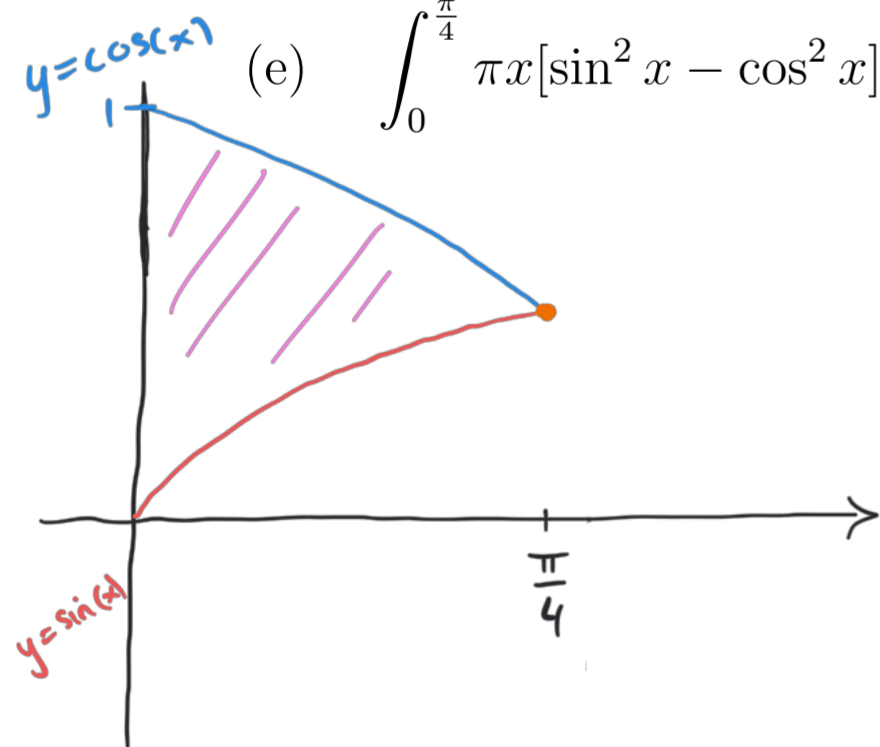
(a) $\int_0^{\frac{\pi}{4}} 2\pi x [\cos^2 x - \sin^2 x] dx$

(b) $\int_0^{\frac{\pi}{4}} 2\pi x [\cos x - \sin x] dx$

(c) $\int_0^{\frac{\pi}{4}} \pi [\sin^2 x - \cos^2 x] dx$

(d) $\int_0^{\frac{\pi}{4}} \pi [\cos^2 x - \sin^2 x] dx$

(e) $\int_0^{\frac{\pi}{4}} \pi x [\sin^2 x - \cos^2 x] dx$



$$V = \int_0^{\pi/4} \pi (\cos^2(x)) dx - \int_0^{\pi/4} \pi \sin^2(x) dx$$

$$= \int_0^{\pi/4} \pi (\cos^2(x) - \sin^2(x)) dx$$

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23.(6 pts.) A force of 10 lbs is required to hold a spring stretched 2 feet beyond its natural length. How much work is done in stretching the spring from its natural length to one foot beyond its natural length?

- (a) 5 ft lb (b) 10 ft lb (c) 2.5 ft lb (d) 1 ft lb (e) 1.5 ft lb

$$\underline{Kx = F} : \quad 2K = 10 \Rightarrow K = 5 \text{ ft-lb/lb}$$

$$W = \int_0^1 Kx dx = 5 \frac{x^2}{2} \Big|_0^1 = \frac{5}{2} \text{ ft-lb}$$

24.(6 pts.) Which integral below gives the volume of the solid generated by rotating the region enclosed by the curves $x = 2 + y$, $x = y^2$ and $y = 0$ about the x -axis.

(a) $\pi \int_0^2 y[(2+y) - y^2] dy$

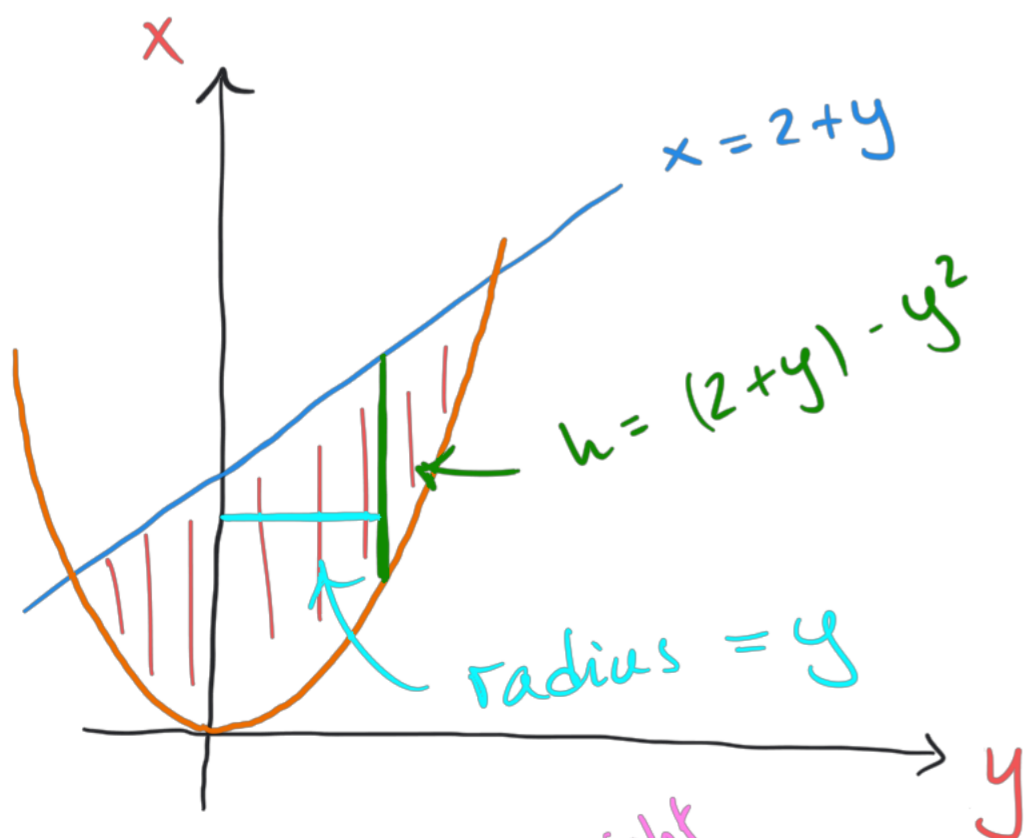
(b) $2\pi \int_0^2 [(2+y)^2 - y^4] dy$

(c) $2\pi \int_0^2 y[(2+y)^2 - y^4] dy$

(d) $2\pi \int_0^2 y[2+y - y^2] dy$

(e) $\pi \int_0^2 y[(2+y)^2 - y^4] dy$

$$\begin{aligned} 2+y &= y^2 \Rightarrow y^2 - y - 2 = 0 \\ &\Rightarrow (y-2)(y+1) = 0 \\ &\Rightarrow y=2 \text{ or } y=-1 \end{aligned}$$



$$V = \int_0^2 2\pi (y) (2+y - y^2) dy$$

Rotating right will cover what rotating left would do.

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25. (6 pts.) Find the average value of $f(x) = 2 \sin x \cos x$ over the interval $[0, \frac{\pi}{4}]$.

- (a) $\frac{\pi}{2}$ (b) $\frac{4}{\pi}$ (c) π (d) $\frac{2}{\pi}$ (e) $\frac{\pi}{4}$

$$f_{\text{ave}} = \frac{1}{(\frac{\pi}{4} - 0)} \int_0^{\pi/4} 2 \sin(x) \cos(x) dx = \frac{4}{\pi} \int_0^{1/\sqrt{2}} 2u du$$

$$= \frac{4}{\pi} u^2 \Big|_0^{1/\sqrt{2}}$$

$$u = \sin(x)$$

$$du = \cos(x) dx$$

$$x=0 \Rightarrow u=0$$

$$x=\pi/4 \Rightarrow u = 1/\sqrt{2}$$

$$= \frac{4}{\pi} \left(\frac{1}{2} - 0 \right)$$

$$= \frac{2}{\pi}$$

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**Math 10550, Final Exam Fall 2014:
December 18, 2016**

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| 8. | (a) | (b) | (c) | (●) | (e) | 22. | (a) | (b) | (c) | (●) | (e) |
| | | | | | | | | | | | |
| 9. | (●) | (b) | (c) | (d) | (e) | 23. | (a) | (b) | (●) | (d) | (e) |
| 10. | (a) | (b) | (c) | (●) | (e) | 24. | (a) | (b) | (c) | (●) | (e) |
| | | | | | | | | | | | |
| 11. | (a) | (b) | (c) | (d) | (●) | 25. | (a) | (b) | (c) | (●) | (e) |
| 12. | (a) | (●) | (c) | (d) | (e) | | | | | | |
| | | | | | | | | | | | |
| 13. | (●) | (b) | (c) | (d) | (e) | | | | | | |
| 14. | (a) | (b) | (●) | (d) | (e) | | | | | | |