

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

**Math 10550, Final Exam:  
December 17, 2008**

- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.
- No calculators are to be used.
- The exam lasts for two hours.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 14 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- |                         |                         |
|-------------------------|-------------------------|
| 1. (a) (b) (c) (d) (e)  | 15. (a) (b) (c) (d) (e) |
| 2. (a) (b) (c) (d) (e)  | 16. (a) (b) (c) (d) (e) |
| .....                   | .....                   |
| 3. (a) (b) (c) (d) (e)  | 17. (a) (b) (c) (d) (e) |
| 4. (a) (b) (c) (d) (e)  | 18. (a) (b) (c) (d) (e) |
| .....                   | .....                   |
| 5. (a) (b) (c) (d) (e)  | 19. (a) (b) (c) (d) (e) |
| 6. (a) (b) (c) (d) (e)  | 20. (a) (b) (c) (d) (e) |
| .....                   | .....                   |
| 7. (a) (b) (c) (d) (e)  | 21. (a) (b) (c) (d) (e) |
| 8. (a) (b) (c) (d) (e)  | 22. (a) (b) (c) (d) (e) |
| .....                   | .....                   |
| 9. (a) (b) (c) (d) (e)  | 23. (a) (b) (c) (d) (e) |
| 10. (a) (b) (c) (d) (e) | 24. (a) (b) (c) (d) (e) |
| .....                   | .....                   |
| 11. (a) (b) (c) (d) (e) | 25. (a) (b) (c) (d) (e) |
| 12. (a) (b) (c) (d) (e) |                         |
| .....                   |                         |
| 13. (a) (b) (c) (d) (e) |                         |
| 14. (a) (b) (c) (d) (e) |                         |

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Multiple Choice

1.(6 pts.) Find the limit

$$\lim_{x \rightarrow 0} \frac{3 - \sqrt{x+9}}{x}$$

(a)  $-\frac{1}{6}$

(b)  $-3$

(c) The limit does not exist.

(d)  $\frac{1}{6}$

(e)  $3$

$$\frac{3 - \sqrt{x+9}}{x} = \frac{3^2 - (\sqrt{x+9})^2}{x(3 + \sqrt{x+9})} = \frac{9 - x - 9}{x(3 + \sqrt{x+9})} = \frac{-x}{x(3 + \sqrt{x+9})} = \frac{-1}{3 + \sqrt{x+9}}$$

$$\lim_{x \rightarrow 0} \frac{3 - \sqrt{x+9}}{x} = \lim_{x \rightarrow 0} \frac{-1}{3 + \sqrt{x+9}} = \frac{-1}{3 + \sqrt{9}} = \frac{-1}{3+3} = \frac{-1}{6}$$

2.(6 pts.) Find all points where the following function is discontinuous

$$f(x) = \begin{cases} \frac{(x-1)(x+2)}{(x^2-1)x} & x \neq 1 \\ \frac{3}{2} & x = 1 \end{cases} \quad \rightarrow \text{"or not defined."}$$

(a)  $x = -2, x = -1, x = 1$

(b)  $x = 0, x = -1$

(c)  $x = 0, x = 1$

(d)  $x = 0, x = -2, x = 1$

(e)  $x = 0, x = -1, x = 1$

$x \neq 1$ :

$$\frac{(x-1)(x+2)}{(x^2-1)x} = \frac{(x-1)(x+2)}{(x-1)(x+1)x}$$

$\leftarrow$  not defined for  $x=0$  or  $x=-1$

$x=1$ :

$$\lim_{x \rightarrow 1} f(x) = \lim_{x \rightarrow 1} \frac{(x-1)(x+2)}{(x-1)(x+1)x} = \lim_{x \rightarrow 1} \frac{x+2}{(x+1)x} = \frac{3}{2(1)} = \frac{3}{2} = f(1)$$

$\uparrow$   
Hence, cts.  
 $\odot x=1$ .

$\uparrow$   
 $x \neq 1$

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3.(6 pts.) If

$$f(x) = \sqrt{1 + \sqrt{1+x}} = \left(1 + (1+x)^{1/2}\right)^{1/2}$$

then  $f'(8) =$

- (a)  $\frac{1}{8}$       (b)  $\frac{1}{9}$       (c)  $\frac{1}{24}$       (d)  $\frac{1}{2}$       (e)  $\frac{1}{12}$

$$f'(x) = \frac{1}{2} \left(1 + (1+x)^{1/2}\right)^{-1/2} \cdot \left(\frac{1}{2}(1+x)^{-1/2} (1)\right)$$

$$f'(8) = \frac{1}{2} \left(1 + \sqrt{9}\right)^{-1/2} \cdot \left(\frac{1}{2} \cdot \frac{1}{\sqrt{9}}\right) = \frac{1}{2} \cdot \frac{1}{\sqrt{4}} \cdot \frac{1}{2} \cdot \frac{1}{3} = \frac{1}{24}$$

4.(6 pts.) The second derivative of

$$f(x) = \frac{\sin x}{x}$$

is

(a)  $\frac{-x^2 \sin x + 4x \cos x + 5 \sin x}{x^3}$

(b)  $\frac{-x^2 \sin x - 3x \cos x + 2 \sin x}{x^3}$

(c)  $\frac{x^2 \sin x + 4x \cos x + 2 \sin x}{x^3}$

(d)  $\frac{-x^2 \sin x - 2x \cos x + 2 \sin x}{x^3}$

(e)  $\frac{-x^2 \sin x - 3x \cos x + 3 \sin x}{x^3}$

$$f'(x) = \frac{x \cos(x) - \sin(x)}{x^2}$$

$$u = x \cos(x) - \sin(x)$$

$$\frac{du}{dx} = \cos(x) - x \sin(x) - \cos(x)$$

$$= -x \sin(x)$$

$$v = x^2 \Rightarrow \frac{dv}{dx} = 2x$$

$$\begin{aligned} \Rightarrow f''(x) &= \frac{x^2 (-x \sin(x)) - (x \cos(x) - \sin(x)) 2x}{x^3} \\ &= \frac{-x^3 \sin(x) - 2x \cos(x) + 2 \sin(x)}{x^3} \end{aligned}$$

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5.(6 pts.) A body travels along a straight line according to the law

$$s = -t^4 - 4t^3 + 20t^2, \quad t \geq 0.$$

At what position, **after** the motion gets started, does the body first come to rest?

- (a)  $s = 36$                       (b)  $s = 24$                       (c)  $s = 2$   
(d)  $s = 32$                       (e)  $s = 12$

$$v(t) = s'(t) = -4t^3 - 12t^2 + 40t = -t(4t^2 + 12t - 40)$$

$v(t) = 0$ : Sol<sup>n</sup> 1:  $t = 0$

OR:  $4t^2 + 12t - 40 = 0 \Leftrightarrow t^2 + 3t - 10 = 0 \Leftrightarrow (t+5)(t-2) = 0$

$\Leftrightarrow \underline{t=2}$  or  $\underline{t=-5}$   
 $\quad \checkmark \qquad \quad \uparrow$  unphysical

$s(2) = -16 - 4(8) + 20(4)$   
 $= -16 - 32 + 80$   
 $= 32$

6.(6 pts.) Find an equation for the tangent line to

$$f(x) = \tan(x^2 + 2x)$$

at the point  $(0, 0)$ .

- (a)  $y = 2x$                       (b)  $y = 0$                       (c)  $y = \sqrt{2}x$   
(d)  $y = 2\sqrt{2}x$                       (e)  $y = -2x$

$$f'(x) = \sec^2(x^2 + 2x) \cdot (2x + 2)$$

$$f'(0) = \sec^2(0) (2(0) + 2) = (1)(2) = 2$$

$$h: y - f(0) = f'(0)(x - 0)$$

$$y - 0 = 2x$$

$$y = 2x$$

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7.(6 pts.) Find an equation for the tangent line to the curve

$$x^3 + y^3 = 4xy$$

at the point (2, 2).

(a)  $y = 2x - 2$

(b)  $y = x$

(c)  $y = -x + 4$

(d)  $y = -x - 4$

(e)  $y = -2x + 6$

$$3x^2 + 3y^2 y' = 4y + 4xy'$$

(2,2)  $\hookrightarrow$   $3(4) + 3(4)y' = 4(2) + 4(2)y'$

$$y - 2 = (-1)(x - 2)$$

$$y = -x + 4$$

$$\Rightarrow 12 + 12y' = 8 + 8y' \Rightarrow 4y' = -4$$

$$\Rightarrow y' = -1$$

8.(6 pts.) The length of a rectangle is increasing at a rate of 8 cm/sec and its width is increasing at a rate of 3 cm/sec. When the length is 20 cm and the width is 10 cm, how fast is the area of the rectangle increasing?

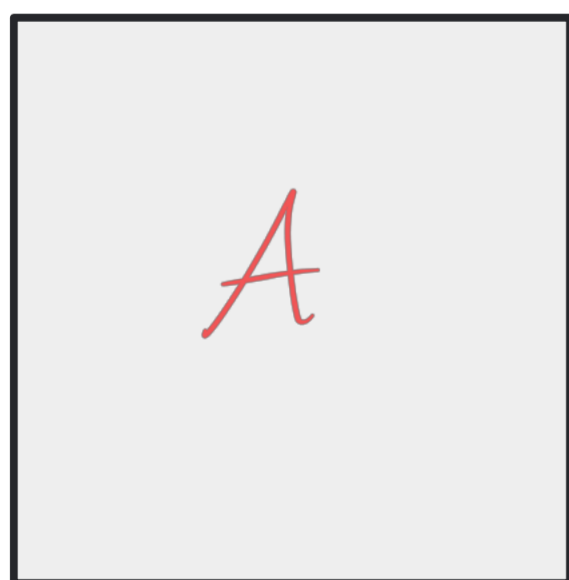
(a)  $140 \text{ cm}^2/\text{sec}$ .

(b)  $211 \text{ cm}^2/\text{sec}$ .

(c)  $190 \text{ cm}^2/\text{sec}$ .

(d)  $11 \text{ cm}^2/\text{sec}$ .

(e)  $24 \text{ cm}^2/\text{sec}$ .



$$\dot{l} = 8 \text{ cm/s}$$

$$\dot{w} = 3 \text{ cm/s}$$

$$A(t) = l(t)w(t)$$

$$\dot{A}(t) = \dot{l}(t)w(t) + l(t)\dot{w}(t)$$

$t=t^*$

$\left( \begin{matrix} 5 \\ \hookrightarrow \end{matrix} \right.$

$$\dot{A}(t^*) = 8w(t^*) + 3l(t^*)$$

$$= 8(10) + 3(20) = 140 \text{ cm}^2/\text{s}$$

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9.(6 pts.) Use linear approximation to estimate

$$\frac{1}{\sqrt{3.9}}$$

(a)  $\frac{1}{\sqrt{3.9}} \approx \frac{9}{20}$

(b)  $\frac{1}{\sqrt{3.9}} \approx \frac{1}{2}$

(c)  $\frac{1}{\sqrt{3.9}} \approx \frac{81}{160}$

(d)  $\frac{1}{\sqrt{3.9}} \approx \frac{11}{20}$

(e)  $\frac{1}{\sqrt{3.9}} \approx \frac{79}{160}$

$$f(x) = \frac{1}{\sqrt{x}} = x^{-1/2}$$

$$f'(x) = -\frac{1}{2} x^{-3/2} = \frac{-1}{2 x^{3/2}}$$

$$\left. \begin{array}{l} f(4) = \frac{1}{2} \\ f'(4) = -\frac{1}{16} \end{array} \right\}$$

$$\left. \begin{array}{l} L_4(x) = \frac{-1}{16}(x-4) + \frac{1}{2} \\ f(3.9) \approx L_4(3.9) \\ = -\frac{1}{16}(3.9-4) + \frac{1}{2} \end{array} \right\}$$

$$= -\frac{1}{16}\left(-\frac{1}{10}\right) + \frac{1}{2}$$

$$= \frac{81}{160}$$

10.(6 pts.) Let

$$f(x) = x^3 + 3x^2 - 24x.$$

Find the absolute maximum and absolute minimum values of  $f$  on the interval  $[0, 10]$ .

(a) Max at  $x = 4$ ; Min at  $x = 0$ .

(b) Max at  $x = 10$ ; Min at  $x = 0$ .

(c) Max at  $x = 4$ ; Min at  $x = 2$ .

(d) Max at  $x = 10$ ; Min at  $x = 2$ .

(e) Max at  $x = 4$ ; Min at  $x = 1$ .

$$f'(x) = 3x^2 + 6x - 24 = 3(x^2 + 2x - 8) = 3(x+4)(x-2)$$

$$f(0) = 0$$

$$f(2) = 8 + 12 - 48 = -28 \quad \leftarrow \text{abs. min.}$$

$$f(4) = 64 + 48 - 72 = 50$$

$$f(10) = 1000 + 300 - 240 = 1060 \quad \leftarrow \text{abs. max}$$

crit. pt's  
x = -4  
x = 2

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11. (6 pts.) Find the local and absolute maximum and minimum of

$$f(x) = 3x^{2/3} - x.$$

- (a) Local min at  $x = 1/8$ ; absolute min at  $x = 1$ ; no absolute max.
- (b) Local min at  $x = 1$ ; local max at  $x = 1/8$ ; no absolute min; absolute max at  $x = -27$ .
- (c) Absolute min at  $x = 0$ ; absolute max at  $x = 8$ .
- (d) Local min at  $x = 0$ ; local max at  $x = 8$ ; no absolute max or min.
- (e) Local max at  $x = 1$ ; no absolute max; absolute min at  $x = 0$ .

$f'(x) = 2x^{-1/3} - 1 = \frac{2}{\sqrt[3]{x}} - 1 = \frac{2 - \sqrt[3]{x^2}}{\sqrt[3]{x}}$

$\leftarrow = 0 \text{ for } x = 8$   
 $\leftarrow \text{not defined for } x = 0$

	$\ominus$	undefined	$\oplus$	0	$\ominus$	
		0		8		$\Rightarrow$ local min @ $x=0$
$2 - \sqrt[3]{x^2}$	$\oplus$	.	$\oplus$	0	$\ominus$	$\cdot$ ) local max @ $x=8$ .
$\sqrt[3]{x}$	$\ominus$	0	$\oplus$		$\oplus$	

12. (6 pts.) Let

$$f(x) = x^{5/3} - 5x^{2/3}.$$

On what intervals is  $f$  concave up?

- (a)  $(-1, 0) \cup (0, \infty)$
- (b)  $(-8, 8)$
- (c)  $(1, \infty)$
- (d)  $(-\infty, -1)$
- (e)  $(0, 8)$

$$f'(x) = \frac{5}{3}x^{2/3} - \frac{10}{3}x^{-1/3}$$

$$f''(x) = \frac{10}{9}x^{-1/3} + \frac{10}{9}x^{-4/3}$$

$$= \frac{10}{9}x^{-1/3} \left(1 + x^{-1}\right)$$

$$= \frac{10}{9} \cdot \frac{1}{\sqrt[3]{x}} \left(1 + \frac{1}{x}\right)$$

	$\ominus$	0	$\oplus$	!	$\oplus$
		-1		0	
$\frac{10}{9}$	$\oplus$		$\oplus$		$\oplus$
$\sqrt[3]{x}$	$\ominus$		$\ominus$	0	$\oplus$
$1 + \frac{1}{x}$	$\oplus$	0	$\ominus$	undefined	$\oplus$

Concave up:  $(-1, 0) \cup (0, \infty)$

$\cdot$ )  $\lim_{x \rightarrow -\infty} f(x) = \infty$

$\Rightarrow$  no absolute max.

$\cdot$ ) no absolute min:

$\lim_{x \rightarrow \infty} f(x) = -\infty$

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13.(6 pts.) Evaluate the limit

$$\lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x} - x).$$

- (a)  $-\infty$       (b) 0      (c) 1      (d) 2      (e)  $\infty$

$$\sqrt{x^2 + 2x} - x = \frac{x^2 + 2x - x^2}{\sqrt{x^2 + 2x} + x} = \frac{2x}{\sqrt{x^2 + 2x} + x} \stackrel{\text{for } x > 0}{=} \frac{2}{\sqrt{1 + \frac{2}{x}} + 1}$$

$$\Rightarrow \lim_{x \rightarrow \infty} (\sqrt{x^2 + 2x} - x) = \frac{2}{\sqrt{1} + 1} = 1$$

14.(6 pts.) The equation of the slant asymptote of the curve  $y = \frac{2x^2 + 1}{x + 1}$  is:

- (a)  $y = 2x$       (b)  $y = 2x - 2$       (c)  $y = -2x + 2$   
(d)  $y = x + 2$       (e)  $y = 2x + 2$

$$x+1 \overline{) 2x^2 + 1} \\ \underline{-2x^2 + 2x} \phantom{+ 1} \\ -2x + 1 \\ \underline{+2x - 2} \\ 3$$



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15.(6 pts.) Suppose the line  $y = 4x - 2$  is tangent to the curve  $y = f(x)$ , when  $x = 1$ . If the Newton's method is used to locate a root of the equation  $f(x) = 0$  and the initial approximation is  $x_1 = 1$ , find the second approximation  $x_2$

- (a) -4      (b) 1      (c) 0      (d) 2      (e)  $\frac{1}{2}$

Tangent line to  $y = f(x)$  when  $x = 1$  :  $y - f(1) = f'(1)(x - 1)$

$$\Rightarrow y = f'(1)(x - 1) + f(1) = f'(1)x - f'(1) + f(1)$$

$$y = 4x - 2$$

$$\Rightarrow f'(1) = 4 \quad \text{and} \quad -f'(1) + f(1) = -2$$

$$\Rightarrow -4 + f(1) = -2 \Rightarrow f(1) = 2$$

$$\Rightarrow x_2 = 1 - \frac{f(1)}{f'(1)} = 1 - \frac{2}{4} = \frac{1}{2}$$

16.(6 pts.) Calculate the following definite integral

$$\int_1^5 (5 - x)^2 dx =$$

- (a) 16      (b)  $-\frac{64}{3}$       (c) 3      (d) -16      (e)  $\frac{64}{3}$

$$\int_1^5 (5 - x)^2 dx = \int_4^0 u^2 (-du) = \int_0^4 u^2 du = \left. \frac{u^3}{3} \right|_0^4 = \frac{64}{3}$$

$$u = 5 - x$$

$$du = -dx$$

$$x = 1 \Rightarrow u = 4$$

$$x = 5 \Rightarrow u = 0$$

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17.(6 pts.) Let  $g(x) = \int_{\sin x}^0 t^2 dt$ . Find  $g'(x)$ .

(a)  $-(\cos x)^2 \cos x$

(b)  $-(\sin x)^2 \cos x$

(c)  $(\cos x)^2 \cos x$

(d)  $-(\sin x)^2 \sin x$

(e)  $(\sin x)^2 \cos x$

$$g'(x) = -(\sin(x))^2 \cdot (\cos(x))$$

$$\frac{d}{dx} \left\{ \int_{g(x)}^a f(t) dt \right\} = -f(g(x)) \cdot g'(x)$$

18.(6 pts.) Calculate the integral  $\int_0^2 \frac{x}{\sqrt{x^2+1}} dx$

(a)  $\sqrt{5} - 1$

(b)  $-\sqrt{5} - 1$

(c)  $1 - \sqrt{5}$

(d)  $\sqrt{5}$

(e) 4

$$\begin{array}{l} u = x^2 + 1 \\ \frac{du}{dx} = 2x \\ \frac{1}{2} du = x dx \\ x = 0 \Rightarrow u = 1 \\ x = 2 \Rightarrow u = 5 \end{array} \quad \left| \quad \int_0^2 \frac{x}{\sqrt{x^2+1}} dx = \int_1^5 \frac{1}{\sqrt{u}} \left(\frac{1}{2}\right) du \right.$$
$$= \frac{1}{2} \int_1^5 u^{-1/2} du$$
$$= \frac{1}{2} \left. \frac{u^{1/2}}{1/2} \right|_1^5$$

10  $= \sqrt{5} - 1$

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19.(6 pts.) Which of the following is a Riemann sum corresponding to the integral

$$\int_0^1 (\tan x + 2) dx. \quad \Delta x = 1/n$$

(a)  $2 + \frac{1}{n} \sum_{i=1}^n \tan\left(\frac{i}{n}\right)$

$$R_n = \sum_{i=1}^n \frac{1}{n} f\left(\frac{i}{n}\right)$$

(b)  $\frac{2}{n} + \frac{2}{n} \sum_{i=1}^n \tan\left(\frac{i}{n}\right)$

$$= \sum_{i=1}^n \frac{1}{n} \left( \tan\left(\frac{i}{n}\right) + 2 \right) \quad \text{(c)}$$

(c)  $\frac{1}{n} \sum_{i=1}^n \left( \tan\left(\frac{i}{n}\right) + 2 \right)$

$$= \sum_{i=1}^n \frac{1}{n} \tan\left(\frac{i}{n}\right) + \sum_{i=1}^n \frac{2}{n}$$

(d)  $\frac{2}{n} \sum_{i=1}^n \tan\left(\frac{2i}{n}\right)$

(e)  $\frac{1}{2n} \sum_{i=1}^n \tan\left(\frac{2i}{n}\right)$

$$= \sum_{i=1}^n \frac{1}{n} \tan\left(\frac{i}{n}\right) + 2 \quad \text{(a)}$$

20.(6 pts.) The point on the line  $6x + y = 9$  that is closest to the origin has  $x$ -coordinate

(a)  $x = \frac{3}{2}$

(b)  $x = 0$

(c)  $x = 1$

(d)  $x = \frac{44}{9}$

(e)  $x = \frac{54}{37}$

$$d = \text{distance to origin} = \sqrt{x^2 + y^2} \Rightarrow D = d^2 = x^2 + y^2$$

$$\Rightarrow D = x^2 + (9 - 6x)^2 = 37x^2 - 108x + 81$$

$$D'(x) = 74x - 108 \Rightarrow \text{critical pt @ } x = \frac{54}{37}$$

$$D''(x) = 74 > 0 \Rightarrow \text{min.}$$

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21. (6 pts.) The curves  $y = x^4 - 3$  and  $y = -x^4 + 5$  enclose an area. Set up a definite integral which calculates the area of this region.

(a)  $\int_{-1}^1 (8 - 2x^4) dx$

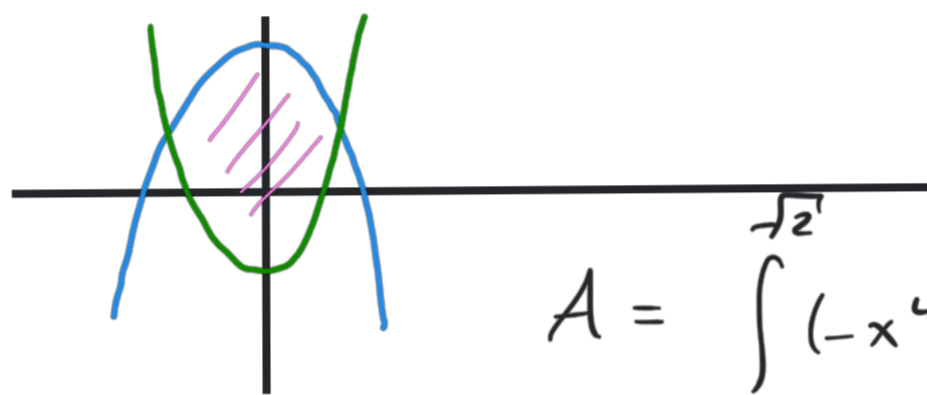
(b)  $\int_0^{\sqrt{3}} (8 - 2x^4) dx$

(c)  $\int_{-1}^1 2 dx$

(d)  $\int_{-\sqrt{2}}^{\sqrt{2}} 2 dx$

(e)  $\int_{-\sqrt{2}}^{\sqrt{2}} (8 - 2x^4) dx$

$$\begin{aligned} x^4 - 3 &= -x^4 + 5 \\ \Rightarrow 2x^4 &= 8 \\ \Rightarrow x^4 &= 4 \\ \Rightarrow x &= \pm\sqrt{2} \end{aligned}$$



$$A = \int_{-\sqrt{2}}^{\sqrt{2}} (-x^4 + 5) - (x^4 - 3) dx$$

22. (6 pts.) The plane region bounded below by the graph of  $y = x$  and above by the graph  $y = \sqrt{x}$  is rotated about the line  $x = 5$ . Which integral below gives the volume?

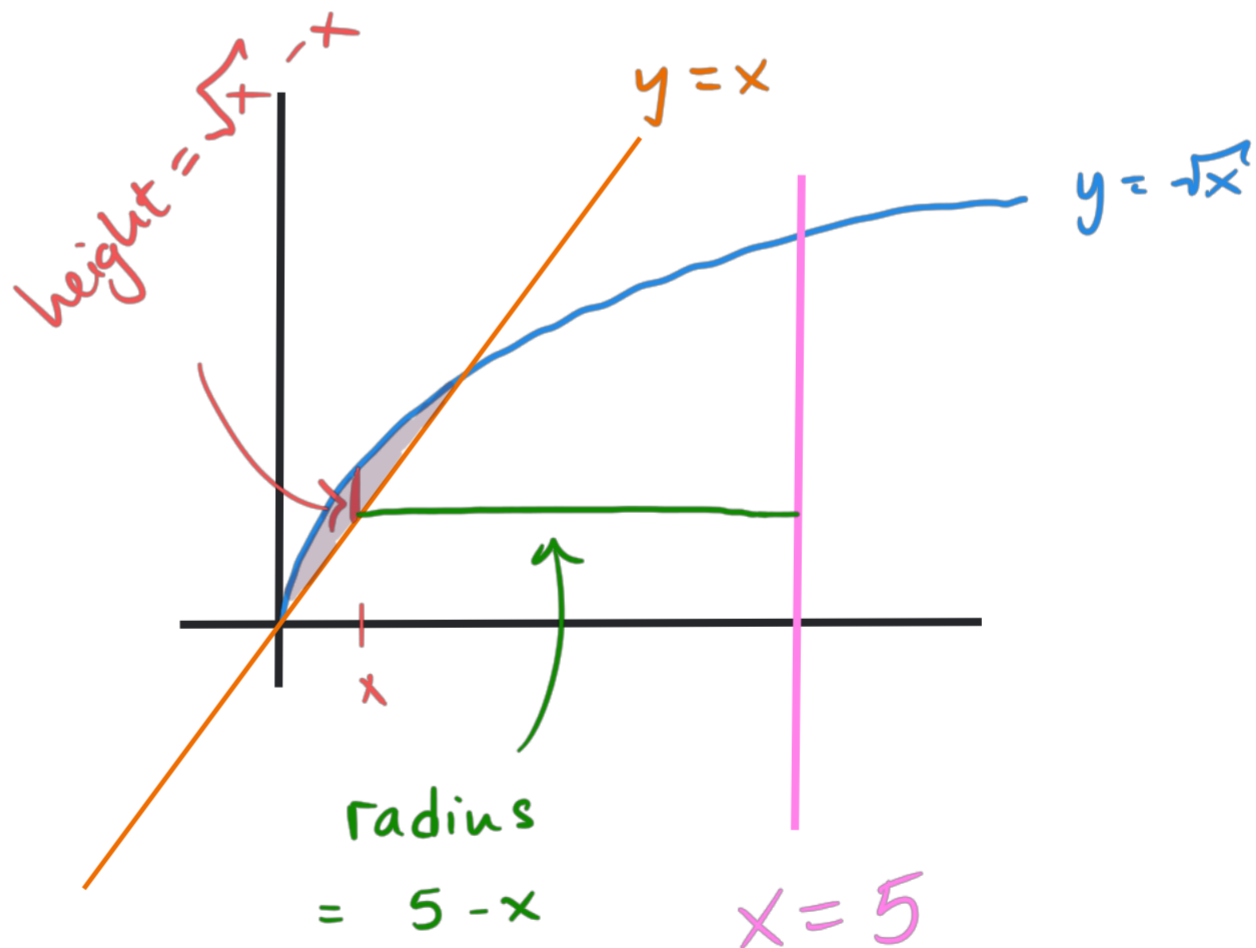
(a)  $\pi \int_0^1 (5 - \sqrt{x})^2 - (5 - x)^2 dx$

(b)  $\pi \int_0^1 (5 - x)^2 - (5 - \sqrt{x})^2 dx$

(c)  $2\pi \int_0^1 (x - 5) \cdot (\sqrt{x} - x) dx$

(d)  $2\pi \int_0^1 (5 - x) \cdot (x - \sqrt{x}) dx$

(e)  $2\pi \int_0^1 (5 - x) \cdot (\sqrt{x} - x) dx$



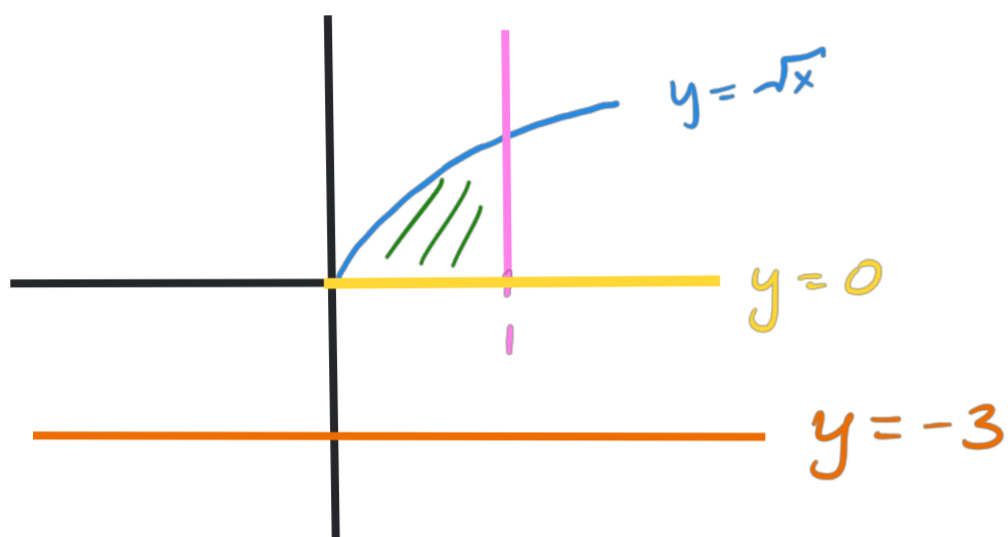
$$V = 2\pi \int_0^1 (\sqrt{x} - x)(5 - x) dx$$

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23. (6 pts.) Consider the plane region bounded by the graphs of  $y = \sqrt{x}$ ,  $y = 0$  and  $x = 1$ . Rotate this region about the line  $y = -3$  and calculate the volume.

- (a)  $\frac{3\pi}{3}$     (b)  $\frac{9\pi}{2}$     (c)  $\frac{7\pi}{2}$     (d)  $\frac{15\pi}{2}$     (e)  $\frac{27\pi}{2}$



$$\begin{aligned}
 V &= \int_0^1 \pi (\sqrt{x} + 3)^2 dx - \int_0^1 \pi (3)^2 dx \\
 &= \pi \int_0^1 (x + 6\sqrt{x}) dx \\
 &= \pi \left( \frac{x^2}{2} + 4x^{3/2} \right) \Big|_0^1 = \pi \left( \frac{1}{2} + 4 \right) = \frac{9\pi}{2}
 \end{aligned}$$

24. (6 pts.) Find the average of  $f(x) = \sin^2(x) \cdot \cos(x)$  over  $[0, \frac{\pi}{2}]$ .

- (a)  $\frac{2}{3\pi}$     (b)  $\frac{2}{\pi}$     (c)  $\frac{1}{3}$   
 (d)  $\frac{1}{\pi}$     (e)  $\frac{1}{3\pi}$

$$\frac{2}{\pi} \int_0^{\pi/2} \sin^2(x) \cos(x) dx = \frac{2}{\pi} \int_0^1 u^2 du = \frac{2}{\pi} \left. \frac{u^3}{3} \right|_0^1$$

$$u = \sin(x)$$

$$\frac{du}{dx} = \cos(x)$$

$$du = \cos(x) dx$$

$$x=0 \Rightarrow u=0$$

$$x=\frac{\pi}{2} \Rightarrow u=1$$

$$= \frac{2}{\pi} \cdot \frac{1}{3} = \frac{2}{3\pi}$$

Name: \_\_\_\_\_

Instructor: \_\_\_\_\_

25. (6 pts.) A (vertical) cylindrical tank has a height 1 meter and base radius 1 meter. It is filled full with a liquid with a density  $100 \text{ kg/m}^3$ . Find the work required to empty the tank by pumping all of the liquid to the top of the tank.

(a)  $0 \text{ kg-m}$

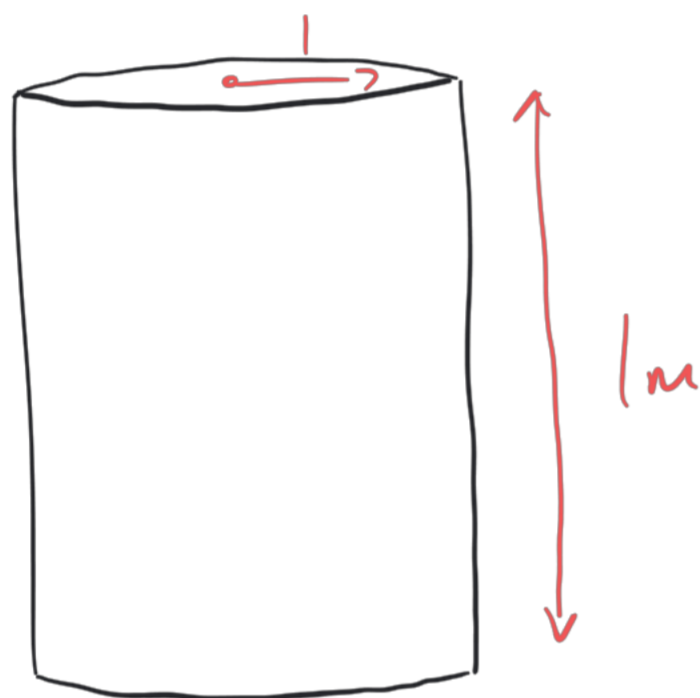
(b)  $200\pi \text{ kg-m}$

(c)  $50\pi \text{ kg-m}$

(d)  $500\pi \text{ kg-m}$

(e)  $100\pi \text{ kg-m}$

Wrong units?  
Missing  $g$ ?



Method 1:

Force needed after  $x$  meters are gone  
= weight left after  $x$  meters are gone

$$= \underbrace{\pi (1)^2 (1-x)}_{\text{Volume}} \underbrace{(100)}_{\text{density}} \underbrace{g}_{\text{gravity}} = 100\pi g (1-x)$$

$$W = \int_0^1 100\pi g (1-x) dx = 100\pi g \left( x - \frac{x^2}{2} \right) \Big|_0^1 = 50\pi g$$

Name: \_\_\_\_\_

Instructor: ANSWERS

**Math 10550, Final Exam:  
December 17, 2008**

- The Honor Code is in effect for this examination, including keeping your answer sheet under cover.
- No calculators are to be used.
- The exam lasts for two hours.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 14 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

- |       |                                  |                                  |                                  |                                  |     |     |                                  |                                  |                                  |     |                                  |
|-------|----------------------------------|----------------------------------|----------------------------------|----------------------------------|-----|-----|----------------------------------|----------------------------------|----------------------------------|-----|----------------------------------|
| 1.    | <input checked="" type="radio"/> | (b)                              | (c)                              | (d)                              | (e) | 15. | (a)                              | (b)                              | (c)                              | (d) | <input checked="" type="radio"/> |
| 2.    | (a)                              | <input checked="" type="radio"/> | (c)                              | (d)                              | (e) | 16. | (a)                              | (b)                              | (c)                              | (d) | <input checked="" type="radio"/> |
| ..... |                                  |                                  |                                  |                                  |     |     |                                  |                                  |                                  |     |                                  |
| 3.    | (a)                              | (b)                              | <input checked="" type="radio"/> | (d)                              | (e) | 17. | (a)                              | <input checked="" type="radio"/> | (c)                              | (d) | (e)                              |
| 4.    | (a)                              | (b)                              | (c)                              | <input checked="" type="radio"/> | (e) | 18. | <input checked="" type="radio"/> | (b)                              | (c)                              | (d) | (e)                              |
| ..... |                                  |                                  |                                  |                                  |     |     |                                  |                                  |                                  |     |                                  |
| 5.    | (a)                              | (b)                              | (c)                              | <input checked="" type="radio"/> | (e) | 19. | (a)                              | (b)                              | <input checked="" type="radio"/> | (d) | (e)                              |
| 6.    | <input checked="" type="radio"/> | (b)                              | (c)                              | (d)                              | (e) | 20. | (a)                              | (b)                              | (c)                              | (d) | <input checked="" type="radio"/> |
| ..... |                                  |                                  |                                  |                                  |     |     |                                  |                                  |                                  |     |                                  |
| 7.    | (a)                              | (b)                              | <input checked="" type="radio"/> | (d)                              | (e) | 21. | (a)                              | (b)                              | (c)                              | (d) | <input checked="" type="radio"/> |
| 8.    | <input checked="" type="radio"/> | (b)                              | (c)                              | (d)                              | (e) | 22. | (a)                              | (b)                              | (c)                              | (d) | <input checked="" type="radio"/> |
| ..... |                                  |                                  |                                  |                                  |     |     |                                  |                                  |                                  |     |                                  |
| 9.    | (a)                              | (b)                              | <input checked="" type="radio"/> | (d)                              | (e) | 23. | (a)                              | <input checked="" type="radio"/> | (c)                              | (d) | (e)                              |
| 10.   | (a)                              | (b)                              | (c)                              | <input checked="" type="radio"/> | (e) | 24. | <input checked="" type="radio"/> | (b)                              | (c)                              | (d) | (e)                              |
| ..... |                                  |                                  |                                  |                                  |     |     |                                  |                                  |                                  |     |                                  |
| 11.   | (a)                              | (b)                              | (c)                              | <input checked="" type="radio"/> | (e) | 25. | (a)                              | (b)                              | <input checked="" type="radio"/> | (d) | (e)                              |
| 12.   | <input checked="" type="radio"/> | (b)                              | (c)                              | (d)                              | (e) |     |                                  |                                  |                                  |     |                                  |
| ..... |                                  |                                  |                                  |                                  |     |     |                                  |                                  |                                  |     |                                  |
| 13.   | (a)                              | (b)                              | <input checked="" type="radio"/> | (d)                              | (e) |     |                                  |                                  |                                  |     |                                  |
| 14.   | (a)                              | <input checked="" type="radio"/> | (c)                              | (d)                              | (e) |     |                                  |                                  |                                  |     |                                  |