Name: ______________________________

Instructor: __________________________

Math 10550, EXAM III
November 17, 3016

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 1 hour and 15 min.
- Be sure that your name is on every page in case pages become detached.
- Be sure that you have all 10 pages of the test.

PLEASE MARK YOUR ANSWERS WITH AN X, not a circle!

1.	(a)	(b)	(c)	(d)	(e)
2.	(a)	(b)	(c)	(d)	(e)
3.	(a)	(b)	(c)	(d)	(e)
4.	(a)	(b)	(c)	(d)	(e)
5.	(a)	(b)	(c)	(d)	(e)
6.	(a)	(b)	(c)	(d)	(e)
7.	(a)	(b)	(c)	(d)	(e)
8.	(a)	(b)	(c)	(d)	(e)
9.	(a)	(b)	(c)	(d)	(e)
10.	(a)	(b)	(c)	(d)	(e)

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Multiple Choice	________
11.	________
12.	________
13.	________
Total	________

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Multiple Choice

1.(6 pts.) The slant asymptote of $y = \dfrac{2x^4 + x^3 + 5}{x^3 - 3x^2 + 2}$ is given by

(a) $y = 2x - 5$ (b) $y = 2x + 4$ (c) $y = 2x + 7$

(d) $y = x + 4$ (e) There are no slant asymptotes.

$$\boxed{2x + 7} + \frac{21x^2 - 4x - 9}{x^3 - 3x^2 + 2}$$

$$x^3 - 3x^2 + 2 \overline{\big)\, 2x^4 + x^3 + 5}$$

$$-\;2x^4 \overset{+}{-} 6x^3 \overset{-}{+} 4x$$

$$7x^3 - 4x + 5$$

$$-\;7x^3 \overset{+}{-} 21x^2 \overset{-}{+} 14$$

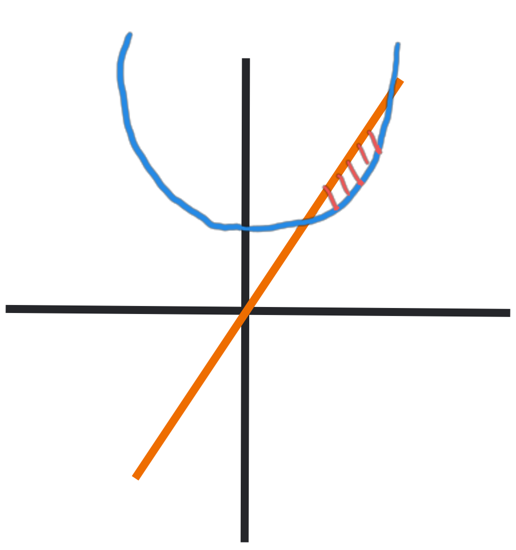
$$21x^2 - 4x - 9$$

2.(6 pts.) Find the area between the curves $y = 4x$ and $y = x^2 + 3$.

(a) $\dfrac{22}{3}$ (b) 0 (c) $\dfrac{1}{3}$ (d) $\dfrac{4}{3}$ (e) 2

$$4x = x^2 + 3 \Rightarrow x^2 - 4x + 3 = 0 \Rightarrow (x-3)(x-1) = 0$$

$$\Rightarrow x = 1 \quad \text{or} \quad x = 3$$



$$\int_1^3 \left(4x - (x^2+3)\right) dx = 2x^2 - \frac{x^3}{3} - 3x \Big|_1^3$$

$$= (18 - 9 - 9) - \left(2 - \tfrac{1}{3} - 3\right) = \frac{4}{3}$$

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3.(6 pts.) If we want to use Newton's method to find an approximate solution to

$$\cos(x) - x = 0$$

with initial approximation $x_1 = \frac{\pi}{2}$, what is x_2?

(a) π (b) $\frac{\pi}{4}$ (c) 0 (d) $\frac{3\pi}{4}$ (e) $\frac{\pi}{2}$

$f(x) = \cos(x) - x$

$f'(x) = -\sin(x) - 1$

$f(\pi/2) = \cos(\frac{\pi}{2}) - \frac{\pi}{2} = -\frac{\pi}{2}$

$f'(\pi/2) = -\sin(\frac{\pi}{2}) - 1 = -2$

$x_1 = \pi/2$

$x_2 = \pi/2 - \frac{f(\pi/2)}{f'(\pi/2)} = \frac{\pi}{2} - \left(\frac{(-\pi/2)}{(-2)}\right)$

$= \frac{\pi}{2} - \frac{\pi}{4} = \frac{\pi}{4}$

4.(6 pts.) A bug being chased by a kitten (both moving in a straight line) enters a kitchen with velocity 1 ft/sec, and accelerates at $\frac{2}{\sqrt{t}}$ft/sec^2. How fast is the bug moving 9 seconds later.

(a) 5 ft/sec (b) 37 ft/sec (c) 13 ft/sec

(d) 4 ft/sec (e) 7 ft/sec

$v(0) = 1$ ft/sec

$a(t) = \frac{2}{\sqrt{t}}$ ft/sec^2

$v'(t) = a(t) = \frac{2}{\sqrt{t}} = 2t^{-1/2}$

$\Rightarrow v(t) = \frac{2t^{1/2}}{1/2} + c_1$

$1 = v(0) = 0 + c_1 \Rightarrow c_1 = 1$

$\Rightarrow v(t) = 4\sqrt{t} + 1$

$\Rightarrow v(9) = 4\sqrt{9} + 1 = 4(3) + 1 = 13$ ft/s

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5.(6 pts.) Which of the following expressions is equal to $\int_0^2 \sin(x^2)dx$?

(a) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \sin\left(\frac{i^2}{n^2}\right)$

(b) $-\cos(4)+1$

(c) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{1}{n} \sin\left(\frac{4i^2}{n^2}\right)$

(d) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \sin\left(\frac{2i}{n}\right)$

(e) $\lim_{n\to\infty} \sum_{i=1}^{n} \frac{2}{n} \sin\left(\frac{4i^2}{n^2}\right)$

$$\int_0^2 \sin(x)\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x\, f(x_i)$$

where $\Delta x = \frac{2-0}{n} = \frac{2}{n}$

$x_i = i\left(\frac{2}{n}\right) = \frac{2i}{n}$

$f(x_i) = \sin\left(\left(\frac{2i}{n}\right)^2\right) = \sin\left(\frac{4i^2}{n^2}\right)$

$$= \lim_{n\to\infty}\left[\sum_{i=1}^{n}\left(\frac{2}{n}\right)\sin\left(\frac{4i^2}{n^2}\right)\right]$$

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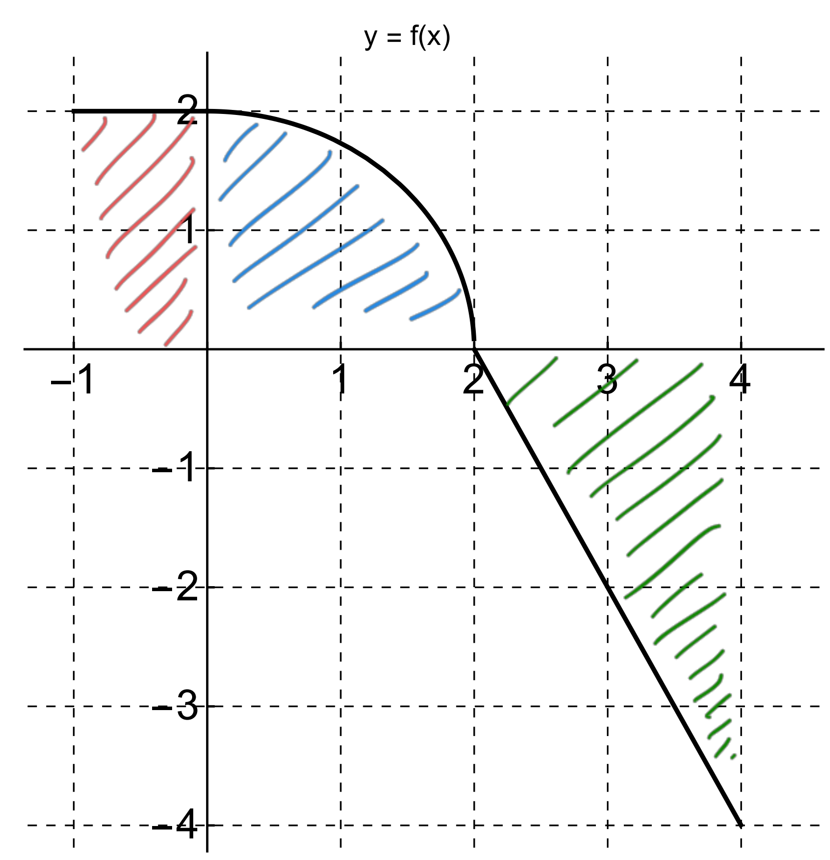
6.(6 pts.) The graph shown below is that of $f(x)$ for $-1 \le x \le 4$ where

$$f(x) = \begin{cases} 2 & \text{if} -1 \le x \le 0 \\ \sqrt{4-x^2} & \text{if} 0 < x \le 2 \\ 4-2x & \text{if} 2 \le x \le 4 \end{cases}$$

Which of the following equals $\int_{-1}^{4} f(x)dx$?



(a) $\pi - 2$ (b) 0 (c) $6 + \pi$

(d) $2\pi - 2$ (e) π

$$\int_{-1}^{4} f(x)\,dx = \quad + \quad -$$

$$= \underbrace{(1)(2)} + \underbrace{\tfrac{1}{4}\left(\pi(2)^2\right)} - \underbrace{\tfrac{1}{2}(2)(4)}$$

$$= 2 + \pi - 4$$

$$= \pi - 2$$

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7.(6 pts.) If $f(x) = \int_{x^3}^{1} \sqrt{1+\sin(t)}\, dt$, then $f'(x) =$

(a) $3x^2\sqrt{1+\sin(x^3)}$ (b) $-\sqrt{1+\sin(x^3)}$ (c) $\sqrt{1+\sin(x)}$

(d) $\sqrt{1+\sin(x^3)}$ (e) $-3x^2\sqrt{1+\sin(x^3)}$

$$f'(x) = -3x^2\sqrt{1+\sin(x^3)}$$

$$\boxed{\frac{d}{dx}\left\{ \int_{g(x)}^{a} f(t)\,dt \right\} = -g'(x)\, f(g(x))}$$

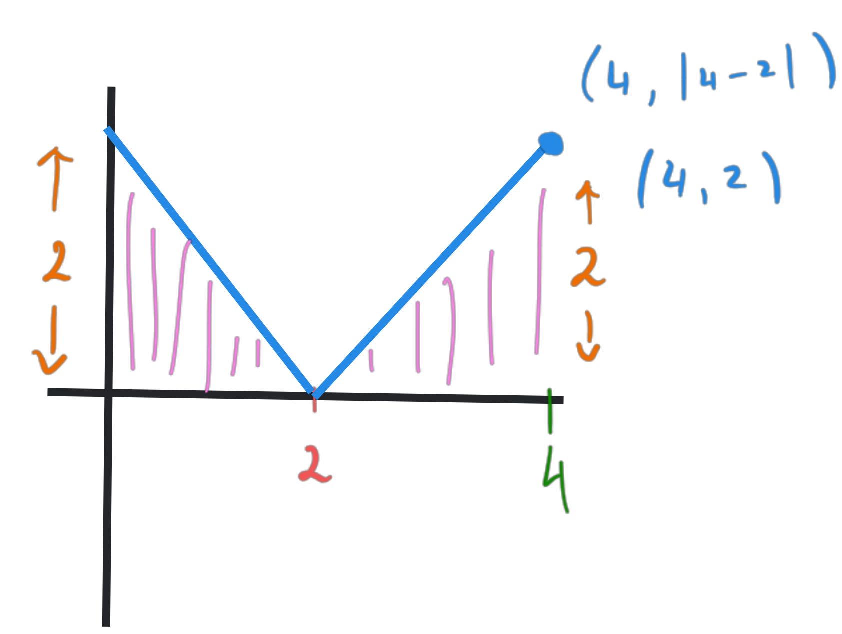
8.(6 pts.) Evaluate $\int_0^4 |x-2| dx$.

(a) -2 (b) 2 (c) 0 (d) 4 (e) 6

$$\int_0^4 |x-2|\,dx = \frac{1}{2}(2)(2) + \frac{1}{2}(2)(2)$$

$$= 2 + 2$$

$$= 4$$



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9.(6 pts.) Evaluate

$$\int \frac{\sin(x^{2/3})}{\sqrt[3]{x}}dx.$$

(a) $\cos(x^{2/3}) + C$

(b) $-\frac{3}{2}\cos(x^{2/3}) + C$

(c) $-\cos(x^{2/3}) + C$

(d) $-\frac{2}{3}\cos(x^{2/3}) + C$

(e) $\frac{3}{2}\cos(x^{2/3}) + C$

$u = x^{2/3}$

$\frac{du}{dx} = \frac{2}{3}x^{-1/3}$

$\frac{3}{2}du = \frac{1}{\sqrt[3]{x}}$

$$\int \frac{\sin(x^{2/3})}{\sqrt[3]{x}}dx = \frac{3}{2}\int \sin(u)\,du$$

$$= -\frac{3}{2}\cos(u) + C$$

$$= -\frac{3}{2}\cos(x^{2/3}) + C$$

10.(6 pts.) Evaluate

$$\int_1^2 \frac{\sec^2(\sqrt{x})}{\sqrt{x}}dx.$$

(a) $\frac{\tan(\sqrt{2})}{2} - \frac{\tan(1)}{2}$

(b) $2\tan(2) - 2\tan(1)$

(c) $2\tan(\sqrt{2}) - 2\tan(1)$

(d) $\frac{\tan(2)}{2} - \frac{\tan(1)}{2}$

(e) $\tan(\sqrt{2}) - \tan(1)$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$2\,du = \frac{1}{\sqrt{x}}dx$

$x = 1 \Rightarrow u = 1$

$x = 2 \Rightarrow u = \sqrt{2}$

$$\int_1^2 \frac{\sec^2(\sqrt{x})}{\sqrt{x}}dx = 2\int_1^{\sqrt{2}} \sec^2(u)\,du$$

$$= 2\tan(u)\Big|_1^{\sqrt{2}}$$

$$7 = 2\tan(\sqrt{2}) - 2\tan(1)$$

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Partial Credit

You must show your work on the partial credit problems to receive credit!

11.(13 pts.) Consider the curve $y^2 - x^2 = 1$ and the point $P(1, 0)$.
(a) Give a formula in terms of x for the distance from the point $P(1, 0)$ to a point (x, y) on the curve $y^2 - x^2 = 1$.

$$d = \sqrt{(x-1)^2 + y^2} = \sqrt{x^2 - 2x + 1 + x^2 + 1}$$

$$= \sqrt{2x^2 - 2x + 2} = \sqrt{2}\sqrt{x^2 - x + 1}$$

(b) Find the point(s) (x, y) on the curve $y^2 - x^2 = 1$ that is(are) closest to the point $P(1, 0)$.

Minimising d is equivalent to minimising $D := d^2$

$D(x) = 2(x^2 - x + 1) \Rightarrow D'(x) = 2(2x - 1)$

Critical pt: $D'(x) = 0 \Rightarrow 2x - 1 = 0 \Rightarrow x = 1/2$

$\Rightarrow y^2 = (\frac{1}{2})^2 + 1 = 5/4 \Rightarrow y = \pm \frac{\sqrt{5}}{2}$

$\Rightarrow P_\pm = (\frac{1}{2}, \pm\frac{\sqrt{5}}{2}) \qquad D(P_\pm) = \sqrt{\frac{1}{4} + \frac{5}{4}} = \sqrt{\frac{3}{2}}$

(c) For full credit for this problem, justify that your answer in part (b) minimizes the distance.

$D'(x) = 2(2x-1)$

	$-$	0 ($\frac{1}{2}$)	$+$
2	$+$		$+$
$2x-1$	$-$	0	$+$

$\Rightarrow x = 1/2$ is a global min.

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12.(13 pts.) A particle traveling on a line accelerates at a rate of $6t$ ft/sec^2.
(a) If the initial velocity is known to be 12 ft/sec, find the distance traveled by the particle in the first second.

$a(t) = 6t$ ft/s^2 $\Rightarrow$ $v(t) = 3t^2 + c_1$

$12 = v(0) = 3(0)^2 + c_1 \Rightarrow c_1 = 12$

$\Rightarrow v(t) = 3t^2 + 12$

Distance travelled in first second : $s(1) = \int_0^1 v(t)\,dt = t^3 + 12t \Big|_0^1 = 1 + 12$
$= 13$ ft/s

(b) If the initial velocity is not known, but the particle is known to travel 20 ft in the first **2** seconds, then find the initial velocity.
(Hint: A definite integral might be useful). Assume $v(0) > 0$.

$v(0) = ? \Rightarrow v(t) = 3t^2 + c_1$

$20 = \int_0^2 v(t)\,dt = \int_0^2 3t^2 + c_1 \,dt = t^3 + c_1 t \Big|_0^2$

$= (8 + 2c_1) - (0)$

$\Rightarrow 2c = 12 \Rightarrow c = 6$

$\Rightarrow v(0) = 3(0)^2 + 6 = 6$ ft/s

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13.(14 pts.) Evaluate the definite integral shown below using right endpoint approximations and the limit definition of the definite integral

$$\int_0^2 \frac{x}{2}\,dx$$

$\left(\text{Note: } 1+2+3+\cdots+n = \sum_{i=1}^{n} i = \frac{n(n+1)}{2}.\right)$ Verify your answer using the fundamental theorem of calculus.

(i) $\displaystyle\int_0^2 \frac{x}{2}\,dx = \lim_{n\to\infty} R_n = \lim_{n\to\infty} \sum_{i=1}^{n} \Delta x\, f(x_i) = \lim_{n\to\infty} \left(\Delta x \sum_{i=1}^{n} f(x_i)\right)$

$\Delta x = \frac{2}{n}$, $x_i = i\left(\frac{2}{n}\right)$, $f(x_i) = \left(\frac{2i}{n}\right)/_2 = \frac{i}{n}$

$$\Rightarrow \int_0^2 \frac{x}{2}\,dx = \lim_{n\to\infty} \frac{2}{n}\left(\sum_{i=1}^{n} \frac{i}{n}\right)$$

$$= \lim_{n\to\infty} \frac{2}{n^2}\left(\sum_{i=1}^{n} i\right)$$

$$= \lim_{n\to\infty} \frac{2}{n^2} \cdot \frac{n(n+1)}{2}$$

$$= \lim_{n\to\infty} \frac{n^2+n}{n^2}$$

$$= 1$$

(ii) $\displaystyle\int_0^2 \frac{x}{2}\,dx = \frac{x^2}{4}\Big|_0^2 = \frac{4}{4} = 1$

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Instructor: ANSWERS

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8.	(a)	(b)	(c)	(•)	(e)
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Please do NOT write in this box.

Multiple Choice ________

11. ________

12. ________

13. ________

Total ________