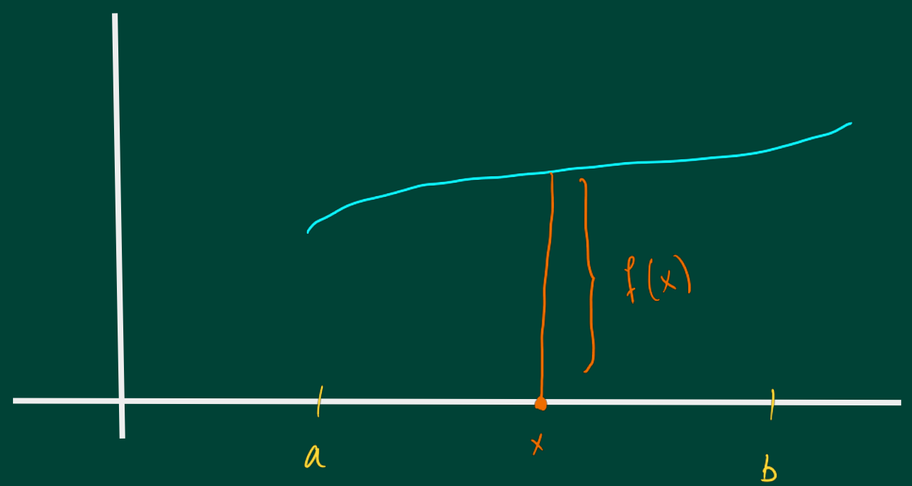
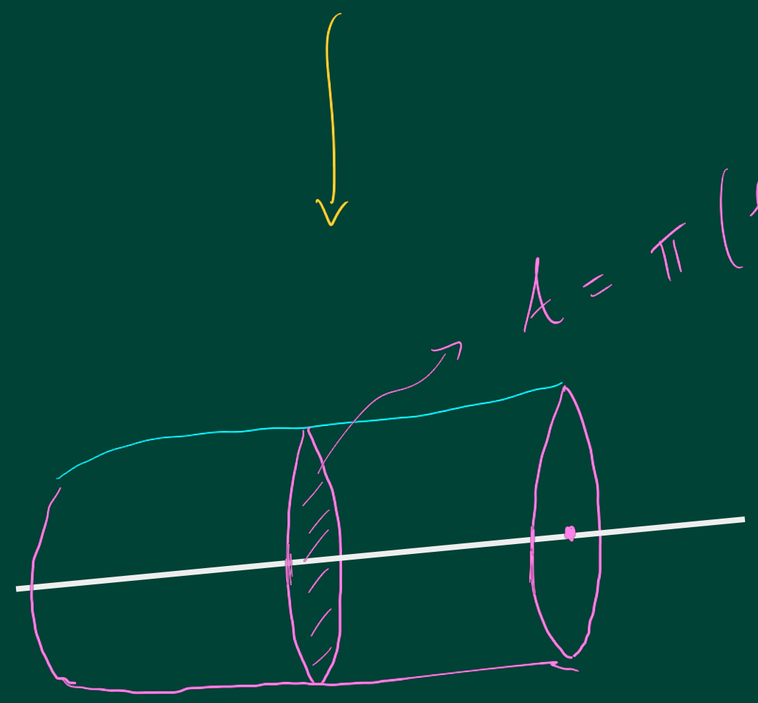
Recall:

1) If $f(x) \geq 0$ for $x \in [a,b]$:



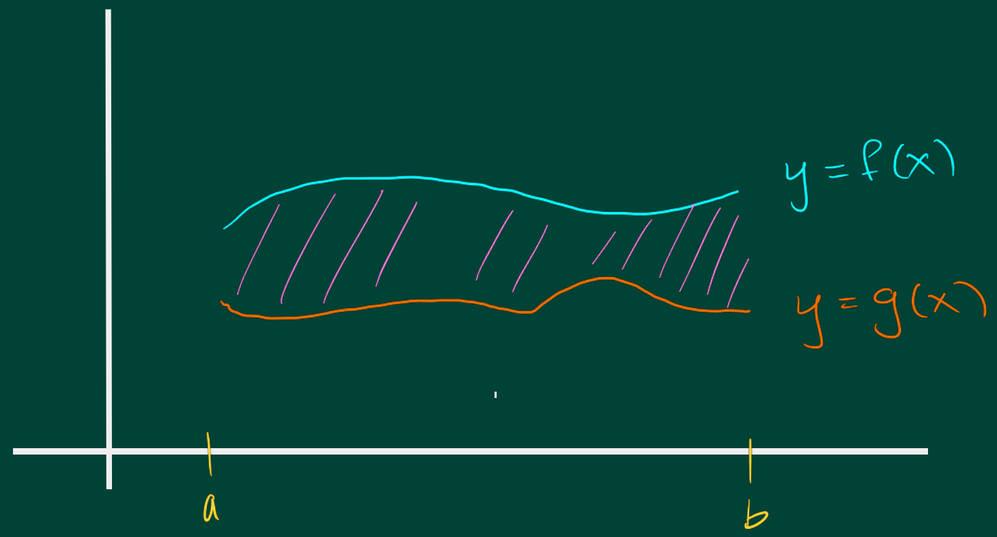
"About x-axis"



"Add up all the disks"

$$V = \int_a^b \pi (f(x))^2 \, dx$$

2)



"About x - axis"

$$V = \int_a^b \pi \left(f(x)\right)^2 dx - \int_a^b \pi \left(g(x)\right)^2 dx$$

$$= \int_a^b \pi \left[f(x)^2 - g(x)^2 \right] dx$$

$$V = 2\pi \int_1^5 x\sqrt{4-(x-3)^2}\,dx$$

$$= 2\pi \int_{-2}^{2} (w+3)\sqrt{4-w^2}\,dw$$

$$= 2\pi \int_{-2}^{2} w\sqrt{4-w^2}\,dw + 6\pi \int_{-2}^{2} \sqrt{4-w^2}\,dw$$

$$= 2\pi \int_{-2}^{2} w\sqrt{4-w^2}\,dw + 6\pi(2\pi)$$

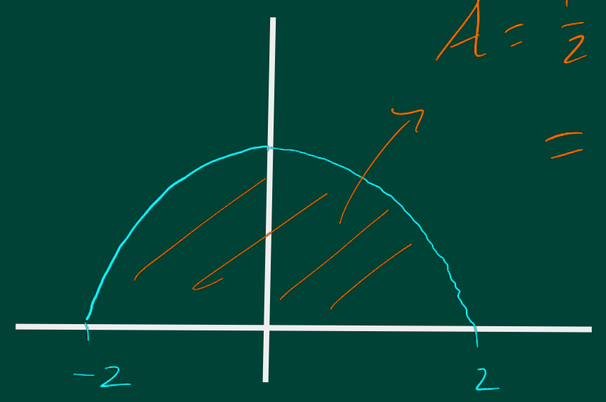
$$= 2\pi \int_{-2}^{2} w\sqrt{4-w^2}\,dw + 12\pi^2$$

$w = (x-3) \Rightarrow w+3 = x$

$dw = dx$

$x = 1 \Rightarrow w = 1-3 = -2$

$x = 5 \Rightarrow w = 5-3 = 2$



$$\int_{-2}^{2} w\sqrt{4-w^2}\,dw$$

$$= \int_{0}^{0} \underbrace{-\frac{1}{2}}\sqrt{u}\,\underbrace{du}$$

$$= 0$$

$u = 4 - w^2 \quad | \quad w = -2 \Rightarrow u = 0$

$\frac{du}{dw} = -2w \quad | \quad w = 2 \Rightarrow u = 0$

$\Rightarrow \quad -\frac{1}{2}\frac{du}{dw} = w$

$\Rightarrow \quad -\frac{1}{2}\,du = \underline{w\,dw}$

OR:

$$\int_{-2}^{2} \underbrace{w\sqrt{4-w^2}}\,dw = 0$$

↑ odd function

Hence, $V = 0 + 12\pi^2 = 12\pi^2$