Exam 3 : • 08:00 – 09:15 , Jordan 105, 15th Nov.

Review Session : Wednesday 7pm – 8pm

DeBartelo 102

Problem Session : Today 4pm – 6pm

(last 3 lectures) 231HH

Tuesday : 09:30 – 11:30

114 Pasquerilla

$$f'(g(x))\,g'(x) = \underbrace{\frac{d}{dx}\left\{ f(g(x)) \right\}}$$

$$\underbrace{\int f'(g(x))\,g'(x)\,dx} = f(g(x)) + C$$

Example:

$$\boxed{\int \underbrace{2x}_{g'(x)} \sin(\underbrace{x^2}_{g(x)})\,dx = -\cos(x^2) + C}$$

$g(x) = x^2$

$f'(x) = \sin(x) \Rightarrow f(x) = -\cos(x) + C$

$g'(x) = 2x$

The Golden Rule of Integration:

- If we are given an integral where we cannot just "write down" an answer, see if one part (part(a)) of the integrand (function you are integrating) differentiates to another part (part(b))

Let $u =$ part(a).

Example:

$$\int 2x\sqrt{x^2+1}\,dx = \int \sqrt{x^2+1}\;2x\,dx = \int \sqrt{u}\;du$$

$u = x^2+1$ (*)

$$\frac{du}{dx} = 2x$$

$$du = \frac{du}{dx}dx = 2x\,dx \quad (*)$$

$$= \int u^{1/2}\,du$$

$$= \frac{u^{3/2}}{3/2} + C$$

$$= \frac{2}{3}u^{3/2} + C$$

$$= \frac{2}{3}(x^2+1)^{3/2} + C$$

$$\boxed{\int_a^b f(g(x))\,g'(x)\,dx = \int_{g(a)}^{g(b)} f(u)\,du}$$

Remark: There are two methods for this kind of problem:

1) "Keep x-limits and sub back in $u = g(x)$ after you integrate".

2) "Change limits and evaluate at new limits".

I personally prefer method 2.

Examples:

1) $$\int_0^1 2x\sqrt{x^2+1}\,dx = \int_1^2 \sqrt{u}\,du = \frac{2}{3}u^{3/2}\Big|_1^2 = \frac{2}{3}\left[2^{3/2} - 1^{3/2}\right]$$

change limits to "u-limits"

$$= \frac{2}{3}\left[\sqrt{8} - 1\right]$$

$u = x^2+1$	$x=0 \Rightarrow u = 0^2+1 = \underline{1}$
$du = 2x\,dx$	$x=1 \Rightarrow u = 1^2+1 = \underline{2}$

NB

2)

$$\int_0^{\pi^2} \frac{\sin(\sqrt{x})}{\sqrt{x}}\,dx = \int_0^{\pi^2} \sin(\sqrt{x})\,\frac{1}{\sqrt{x}}\,dx = \int_0^{\pi} \sin(u)\,2du$$

$u = \sqrt{x}$

$\frac{du}{dx} = \frac{1}{2\sqrt{x}}$

$du = \frac{du}{dx}\,dx$

$2du = \frac{1}{\sqrt{x}}\,dx$

$x = 0 \Rightarrow u = \sqrt{0} = 0$

$x = \pi^2 \Rightarrow u = \sqrt{\pi^2} = \pi$

$$= 2\int_0^{\pi} \sin(u)\,du = -2\cos(u)\Big|_0^{\pi}$$

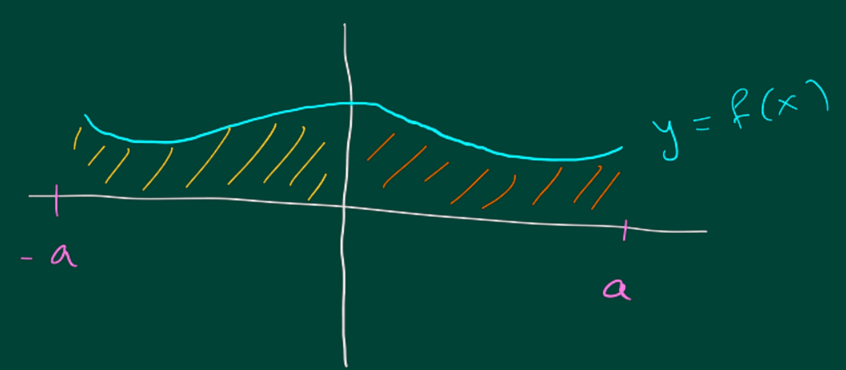
$$= -2\left[\cos(\pi) - \cos(0)\right]$$

$$= -2\left[-1 - 1\right]$$

$$= 4$$

Even and Odd Functions:

Even



If $f(x)$ is even, it's graph is symmetric about the y-axis.

Hence $=$

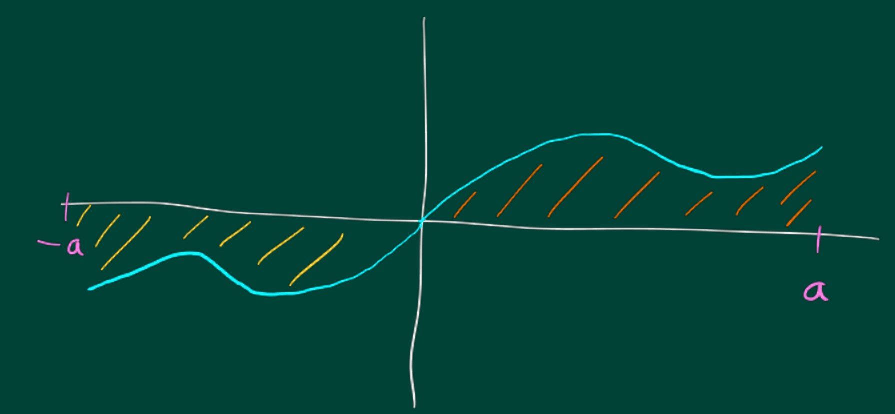
Hence,

$$\int_{-a}^{a} f(x)\,dx = \quad + $$

$$= 2$$

$$= 2\int_{0}^{a} f(x)\,dx$$

Odd



If $f(x)$ is odd, its graph is antisymmetric about the y-axis.

Hence, $= -$

Hence,

$$\int_{-a}^{a} f(x)\,dx = \quad + $$

$$= - \quad + $$

$$= 0$$

Example:

$$\int_{-\frac{\pi}{4}}^{\pi/4} \tan^5(x)\,dx = 0$$

$f(x) = \tan^5(x)$ is odd.

As $f(-x) = \dfrac{\sin^5(-x)}{\cos^5(-x)} = \dfrac{-\sin^5(x)}{\cos^5(x)} = -\tan^5(x)$ (*)

Example: $\int \sin(2x+1)\,dx = \frac{1}{2}\int \sin(u)\,du$

"Linear substitution"

$u = 2x+1$

$du = 2\,dx$

$\frac{1}{2}du = dx$

$= -\frac{1}{2}\cos(u) + C$

$= -\frac{1}{2}\cos(2x+1) + C$