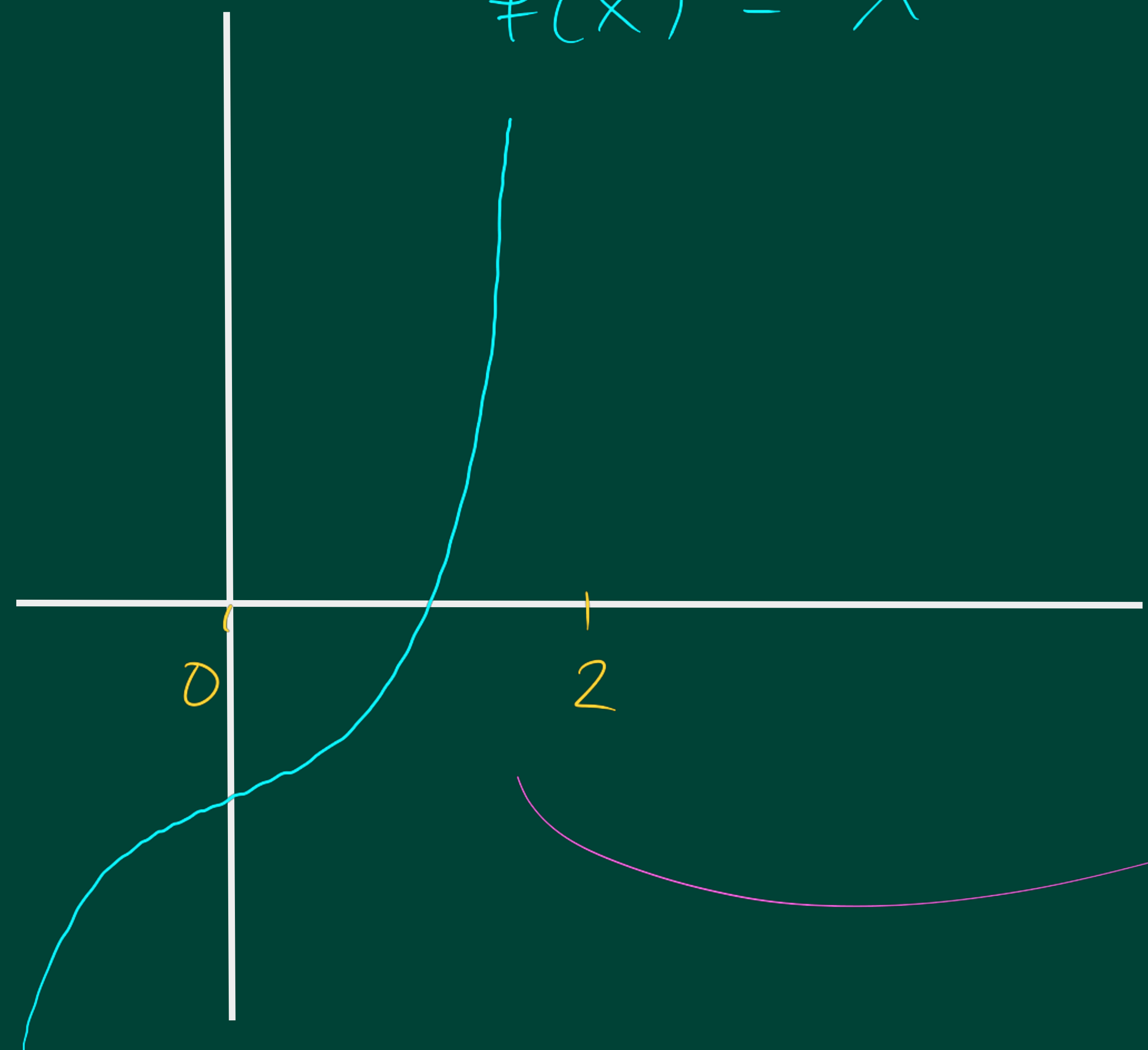
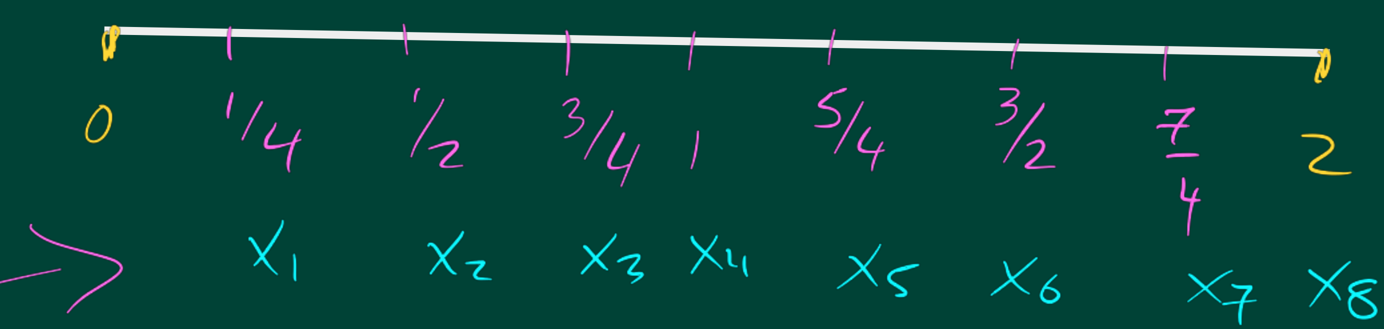
1) $f(x) = x^3 - 1$

Example : Page 2





$$\Delta x = \frac{b-a}{n} = \frac{2}{8} = \frac{1}{4}$$

$n = 8$, $b = 2$, $a = 0$

$x_1 = 1/4$	$x_4 = 1$	$x_7 = 7/4$
$x_2 = 1/2$	$x_5 = 5/4$	
$x_3 = 3/4$	$x_6 = 3/2$	$x_8 = 2$

$$f(x_1) = f(1/4) = \left(\frac{1}{4}\right)^3 - 1 = \frac{1}{64} - 1 = \frac{-63}{64}$$

$$f(x_2) = f(1/2) = \frac{-7}{8}$$

$$f(x_3) = f(3/4) = \frac{-37}{64}$$

$$f(x_4) = f(1) = 0$$

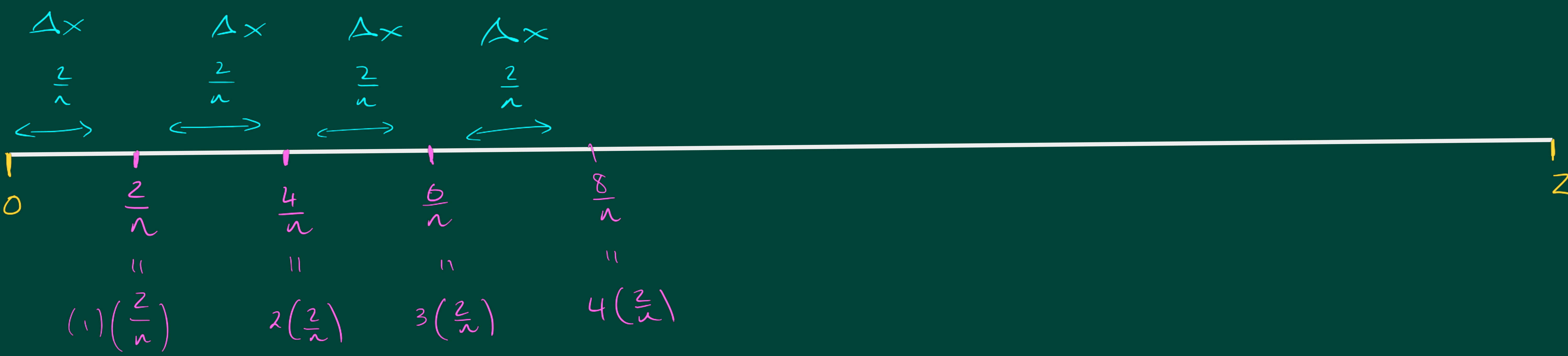
$$f(x_5) = f(5/4) = \frac{61}{64}$$

$$f(x_6) = f(3/2) = 19/8$$

$$f(x_7) = f(7/4) = \frac{279}{64}$$

$$f(x_8) = f(2) = 7$$

2) $\Delta x = \frac{2}{n} \leftarrow$ chop interval into n pieces. This is the length of each piece.



$x_1 = \frac{2}{n}$, $x_2 = \frac{4}{n} = 2\left(\frac{2}{n}\right)$, $x_3 = \frac{6}{n} = 3\left(\frac{2}{n}\right)$, . . .

$$\boxed{x_i = i\left(\frac{2}{n}\right), \quad \text{for} \quad 1 \le i \le n.}$$

Recall: $R_n = \Delta x \left(\underbrace{f(x_1)} + \underbrace{f(x_2)} + \cdots + \underbrace{f(x_n)} \right)$

need to find these

$$f(x) = x^3 - 1$$

$$f(x_1) = (x_1)^3 - 1 = \left((1)\left(\frac{2}{n}\right)\right)^3 - 1 = (1)^3\left(\frac{2}{n}\right)^3 - 1$$

$$f(x_2) = (x_2)^3 - 1 = \left((2)\left(\frac{2}{n}\right)\right)^3 - 1 = (2)^3\left(\frac{2}{n}\right)^3 - 1$$

$$f(x_3) = (x_3)^3 - 1 = \left((3)\left(\frac{2}{n}\right)\right)^3 - 1 = (3)^3\left(\frac{2}{n}\right)^3 - 1$$

$$\vdots$$

$$f(x_i) = (x_i)^3 - 1 = \left((i)\left(\frac{2}{n}\right)\right)^3 - 1 = (i)^3\left(\frac{2}{n}\right)^3 - 1$$

$$\vdots$$

$$f(x_n) = (x_n)^3 - 1 = \left((n)\left(\frac{2}{n}\right)\right)^3 - 1 = (n)^3\left(\frac{2}{n}\right)^3 - 1$$

$$R_n = \Delta x \left(f(x_1) + f(x_2) + \cdots + f(x_n) \right)$$

$$= \left(\frac{2}{n}\right)\left[\left((1)^3\left(\frac{2}{n}\right)^3 - \underline{1} \right) + \left((2)^3\left(\frac{2}{n}\right)^3 - \underline{1} \right) \right.$$

$$+ \left((3)^3\left(\frac{2}{n}\right)^3 - \underline{1} \right) + \cdots + \left((i)^3\left(\frac{2}{n}\right)^3 - \underline{1} \right)$$

$$\left. + \left((n)^3\left(\frac{2}{n}\right)^3 - \underline{1} \right) \right]$$

$$= \left(\frac{2}{n}\right)\left[(1)^3\left(\frac{2}{n}\right)^3 + (2)^3\left(\frac{2}{n}\right)^3 + \cdots + (n)^3\left(\frac{2}{n}\right)^3 - \underline{n} \right]$$

See (*) at end for Σ notation

collected all the -1's

$$R_n = \left(\frac{2}{n}\right)\left[\left(\frac{2}{n}\right)^3 \underbrace{\left\{(1)^3 + (2)^3 + \cdots + (n)^3\right\}} - n\right]$$

$$\boxed{1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2}$$

$$= \left(\frac{2}{n}\right)\left[\left(\frac{2}{n}\right)^3 \underbrace{\left\{\left(\frac{n(n+1)}{2}\right)^2\right\}} - n\right]$$

$$= \left(\frac{2}{n}\right)^4 \left\{\left(\frac{n(n+1)}{2}\right)^2\right\} - \left(\frac{2}{n}\right)n$$

$$= \frac{16}{n^4}\left(\frac{n^2(n+1)^2}{4}\right) - 2$$

$$= 4\left(\frac{n^2(n+1)^2}{n^4}\right) - 2$$

3) $$\int_0^2 f(x)\,dx = \int_0^2 (x^3-1)\,dx = \lim_{n\to\infty} R_n$$

$$\lim_{n\to\infty} R_n = \lim_{n\to\infty}\left(4\left(\frac{n^2(n+1)^2}{n^4}\right) - 2\right)$$

$$= \lim_{n\to\infty}\left(4\left(\frac{n^2(n^2+2n+1)}{n^4}\right) - 2\right)$$

$$= \lim_{n\to\infty}\left(4\left(\frac{n^4+2n^3+n^2}{n^4}\right) - 2\right)$$

$$= 4 - 2$$

$$= 2$$

Evaluate $\underline{\text{after}}$ finding antiderivative.

Remark: $$\underbrace{\int f(x)\,dx}_{\text{no limits : Antiderivative!}}\Big|_0^2 = \frac{x^4}{4} - x\Big|_0^2 = \frac{16}{4} - 2 = 2$$

HMMM...

(*) Using Σ notation: (faster)

$$R_n = \sum_{i=1}^{n} \Delta x \, f(x_i) = \sum_{i=1}^{n} \left(\frac{2}{n}\right)\left((i)^3 \left(\frac{2}{n}\right)^3 - 1 \right)$$

$$= \sum_{i=1}^{n} \left\{ (i)^3 \left(\frac{2}{n}\right)^4 - \left(\frac{2}{n}\right) \right\}$$

$$= \sum_{i=1}^{n} (i)^3 \left(\frac{2}{n}\right)^4 - \sum_{i=1}^{n} \left(\frac{2}{n}\right)$$

$$= \left(\frac{2}{n}\right)^4 \underbrace{\sum_{i=1}^{n} (i)^3} - \left(\frac{2}{n}\right) \underbrace{\sum_{i=1}^{n} 1}$$

$$\boxed{\sum_{k=1}^{m} 1 = \underbrace{1 + \dots + 1}_{m \text{ times}} = m}$$

$$= \left(\frac{2}{n}\right)^4 \underbrace{\left(\frac{n(n+1)}{2}\right)^2} - \left(\frac{2}{n}\right) \underbrace{(n)}$$

$$= \left(\frac{2}{n}\right)^4 \left(\frac{n(n+1)}{2}\right)^2 - 2$$