Office Hours: 5pm - 6pm B26 HH

Review Session: 7pm - 8pm

102 DBRT

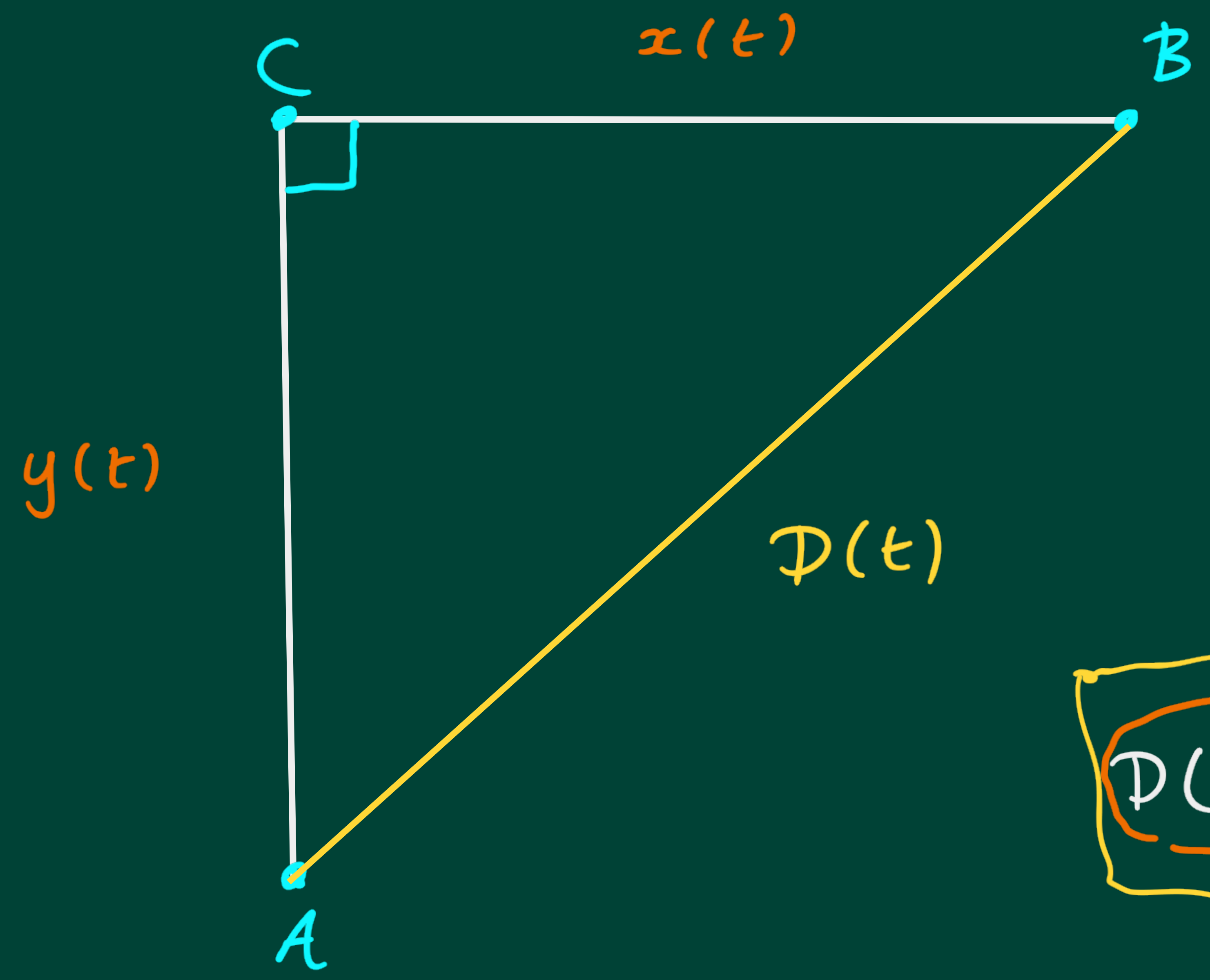
Prof. Pugh.

Exam: 08:00am - 09:15am

105 Jordan

Moving Picture

11. **Given:**



$t \to \boxed{D} \to D(t)$

$\hookrightarrow \boxed{(\cdot)^2} \to D(t)^2$

$\dot{y}(t) = -2$ mph

$\dot{x}(t) = 3$ mph

$$\boxed{D(t)^2 = x(t)^2 + y(t)^2}$$

Want: $\dot{D}$ @ $y = 4$ and $x = 3$ @ $t = t_*$

Sol^n:

$$2D(t)\dot{D}(t) = 2x(t)\dot{x}(t) + 2y(t)\dot{y}(t)$$

$$D(t)\dot{D}(t) = x(t)\dot{x}(t) + y(t)\dot{y}(t)$$

$t = t_*$

$$D(t_*)\dot{D}(t_*) = x(t_*)\dot{x}(t_*) + y(t_*)\dot{y}(t_*)$$

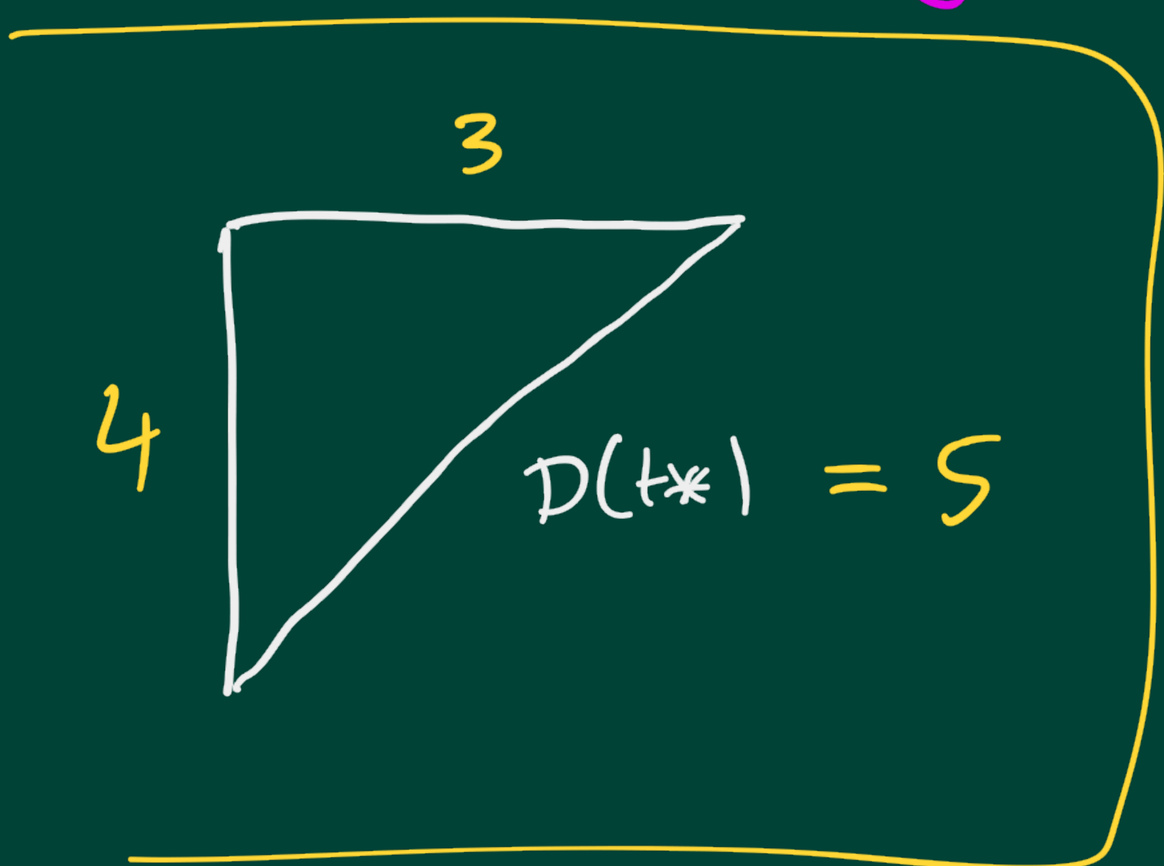
$$\underbrace{D(t_*)}_{*}\,\dot{D}(t_*) = x(t_*)\,\dot{x}(t_*) + y(t_*)\,\dot{y}(t_*)$$

$$= 3(3) + 4(-2)$$

$$= 9 - 8$$

$$= 1$$

"Instant Picture"



$$\Rightarrow \quad 5\,\dot{D}(t_*) = 1$$

$$\dot{D}(t_*) = \frac{1}{5} \text{ mph}$$

12./ $f(x) = x^3 + x - \frac{1}{x}$, Domain $= (0,\infty)$

"**Remark:** f is differentiable on $(0,\infty)$ as it is a polynomial − rational (defined on $(0,\infty)$)". (*)

- $f(1) = 1^3 + 1 - \frac{1}{1} = 1$

  $f(\frac{1}{2}) = (\frac{1}{2})^3 + \frac{1}{2} - 1/\frac{1}{2} = \frac{1}{8} + \frac{1}{2} - 2 = \frac{-11}{8}$

  < 0

Hence, by (*) and IVT, there is a $c \in (\frac{1}{2}, 1)$ such that $f(c) = 0$.

Hence, we have at least one sol$^{\underline{n}}$.

$$f'(x) = 3x^2 + 1 + \frac{1}{x^2} > 0$$

Hence, by (*) and Rolle's Thm, we cannot have a $d \neq c$ such that $f(d) = 0$, as otherwise, there would be an e between c and d such that $f'(e) = 0$.

Hence, there is exactly one sol^n in $(0, \infty)$.

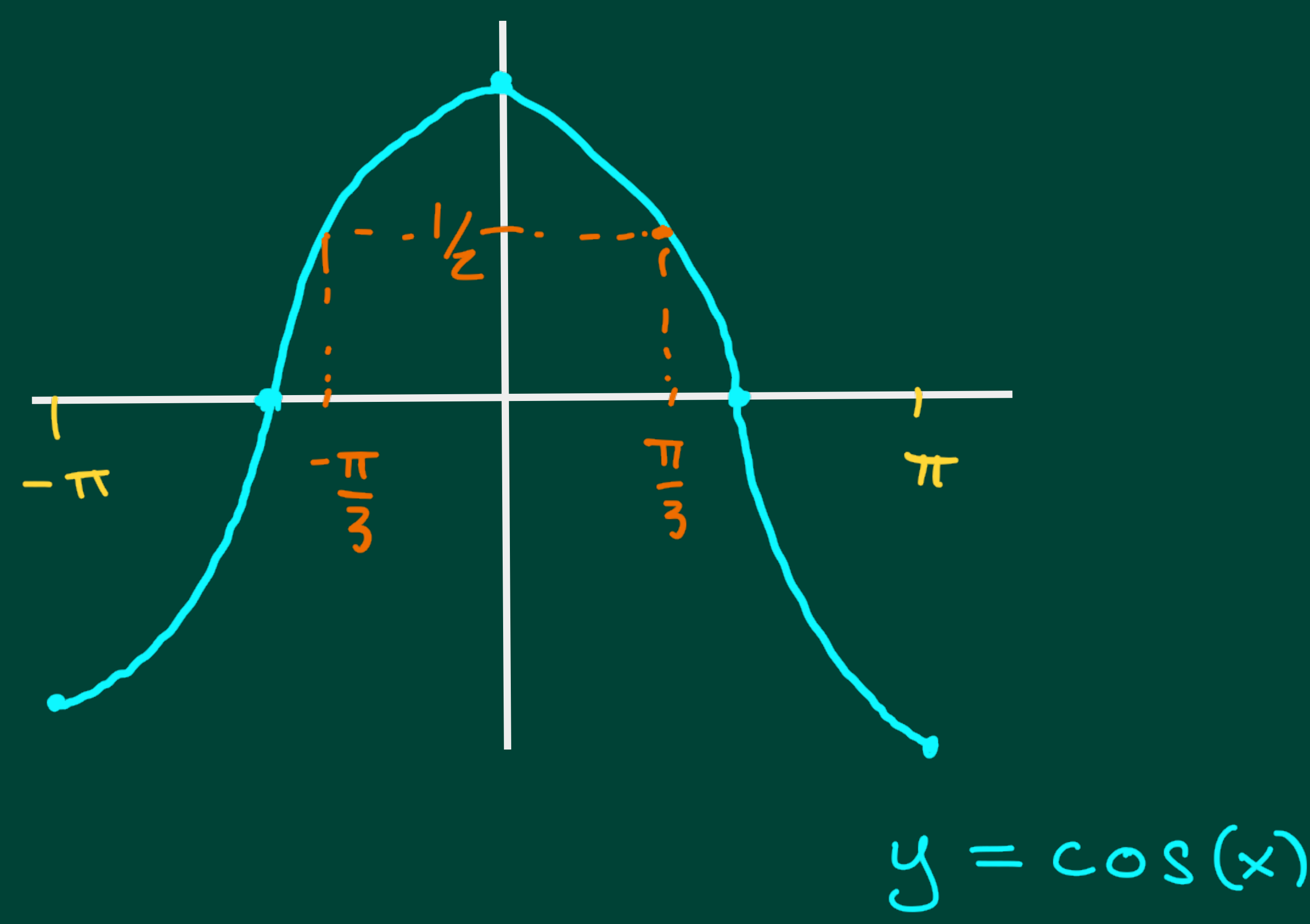
13. $f(x) = x - \sin(2x)$ with domain $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

(a) $f'(x) = 1 - 2\cos(2x)$ ← defined for all $x \in \left(-\frac{\pi}{2}, \pi/2\right)$

$f'(x) = 0$

$1 - 2\cos(2x) = 0$

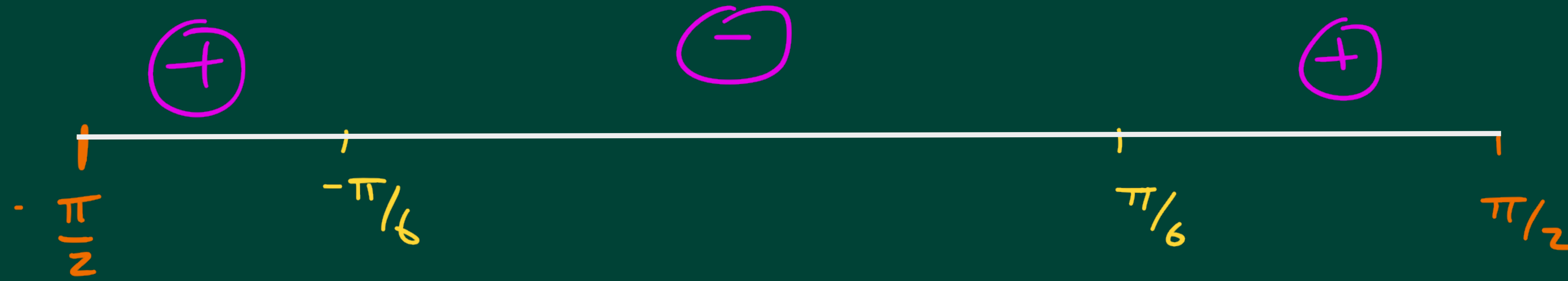
$\frac{1}{2} = \cos(2x)$

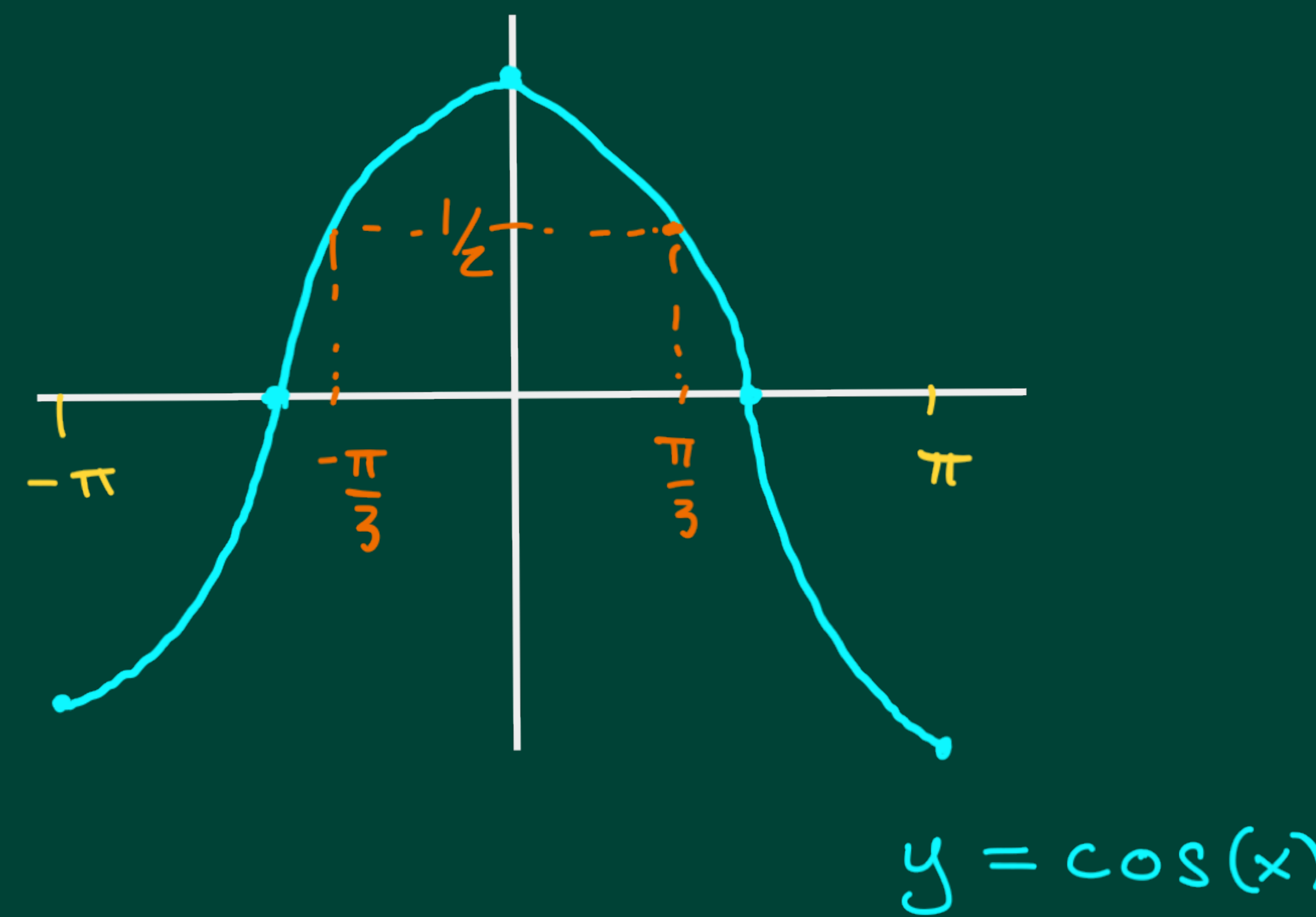


$2x = \pm \frac{\pi}{3}$

$x = \pm \pi/6$

Critical pts: $\pi/6$, $-\pi/6$





$1 - 2\cos(2x)$

Increasing: $\left(-\frac{\pi}{2}, -\frac{\pi}{6}\right) \cup \left(\frac{\pi}{6}, \frac{\pi}{2}\right)$

Decreasing: $\left(\frac{-\pi}{6}, \pi/6\right)$

(c) local max @ $-\pi/6$ (Derivative changes from + to −).

local min @ $\pi/6$ (Derivative changes from − to +).

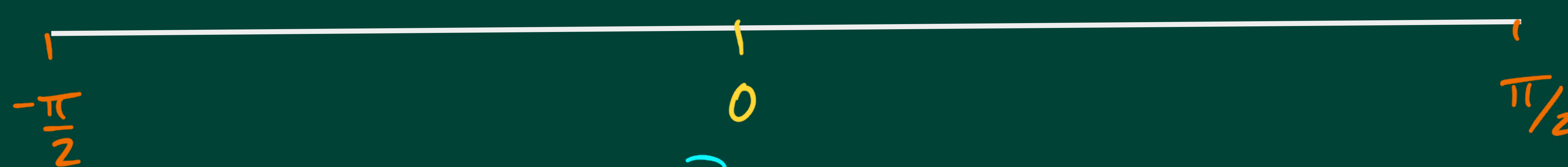
$f''(x) = \underbrace{4\sin(2x)}$ ← need where this is positive and where it's negative.

concave down on $\left(-\frac{\pi}{2}, 0\right)$

$-$

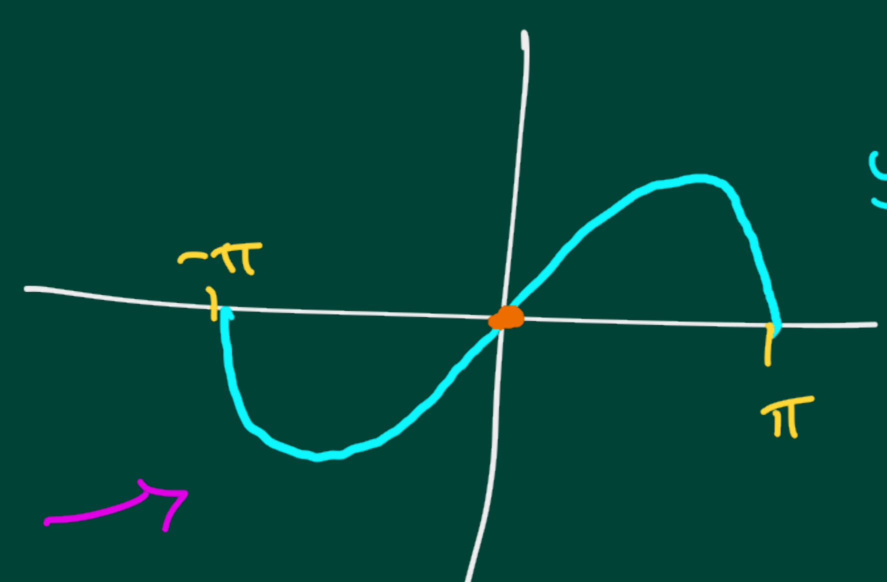
concave up on $\left(0, \pi/2\right)$

$+$



?

Inflection pt @ $x = 0$



$y = \sin(x)$ ← pos.

neg. →

Inflection pt : $(0, f(0)) = (0, 0)$

5./ $f(x) = \sqrt[3]{x}$ @ $a = -8$

$$L_a(x) = f'(a)(x-a) + f(a)$$

$$f(a) = \sqrt[3]{-8} = -2$$

$$f'(x) = \frac{1}{3} x^{-2/3} \qquad f'(-8) = \frac{1}{3} \frac{1}{(-8)^{2/3}}$$

$$= \frac{1}{3} \frac{1}{(-2)^2} = \frac{1}{12}$$

$$L_{-8}(x) = \frac{1}{12}(x+8) - 2$$

$$\sqrt[3]{-8.12} = f(-8.12) \approx L_8(-8.12)$$

$$= \frac{1}{12}(-8.12+8) - 2$$

$$= \frac{1}{12}(-0.12) - 2$$

$$= -0.01 - 2$$

$$= -2.01$$