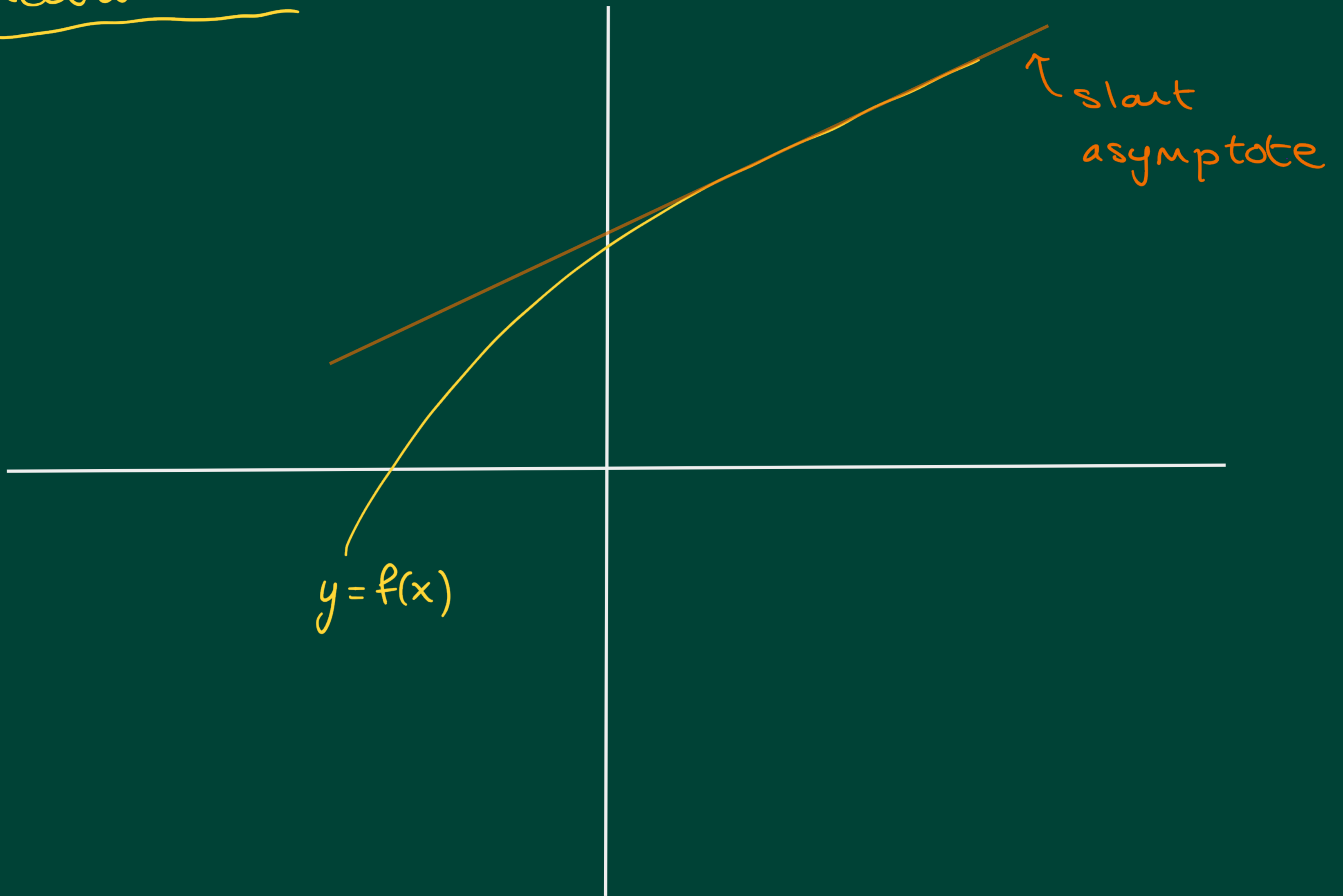
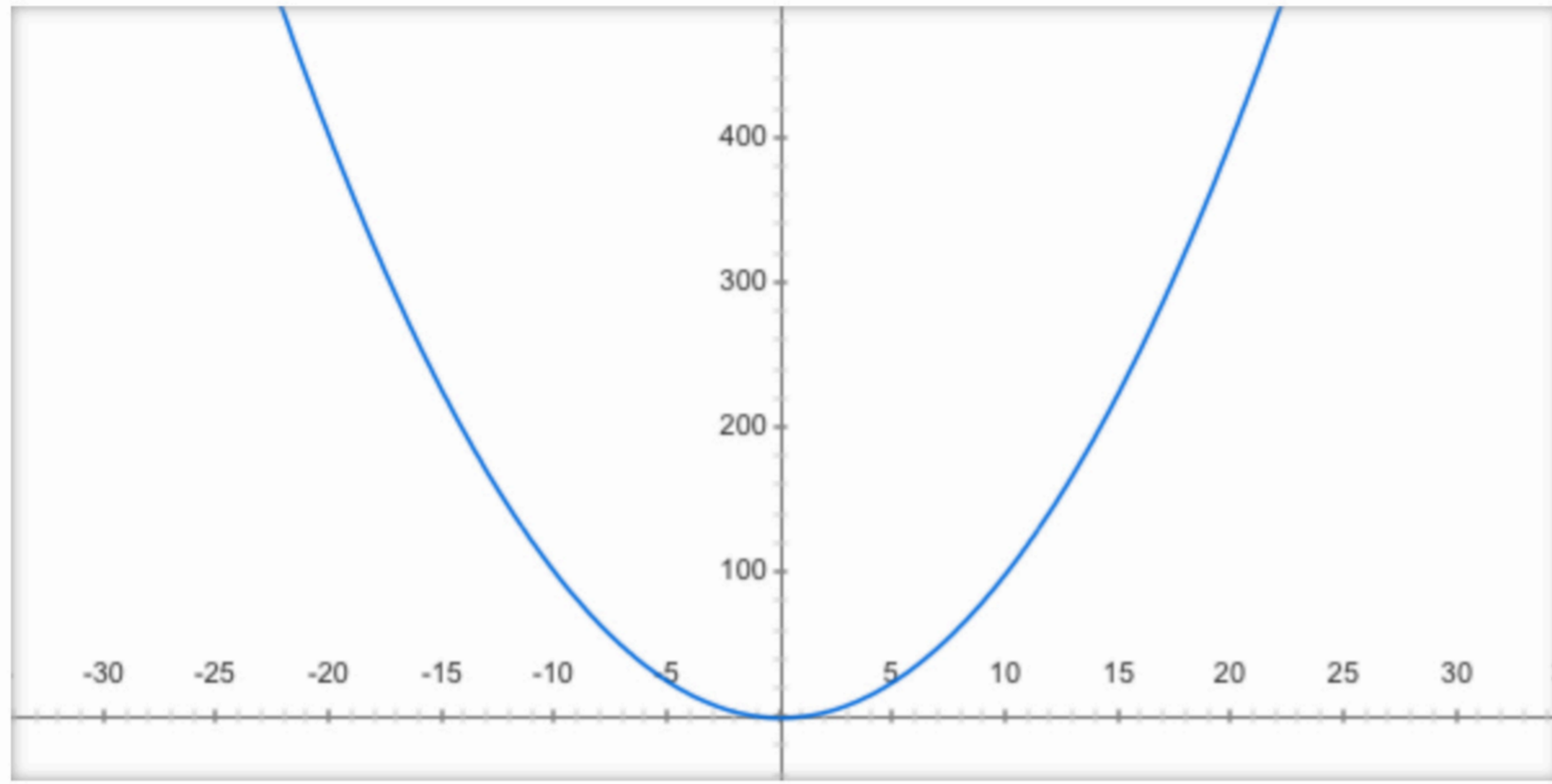


Lecture 20:

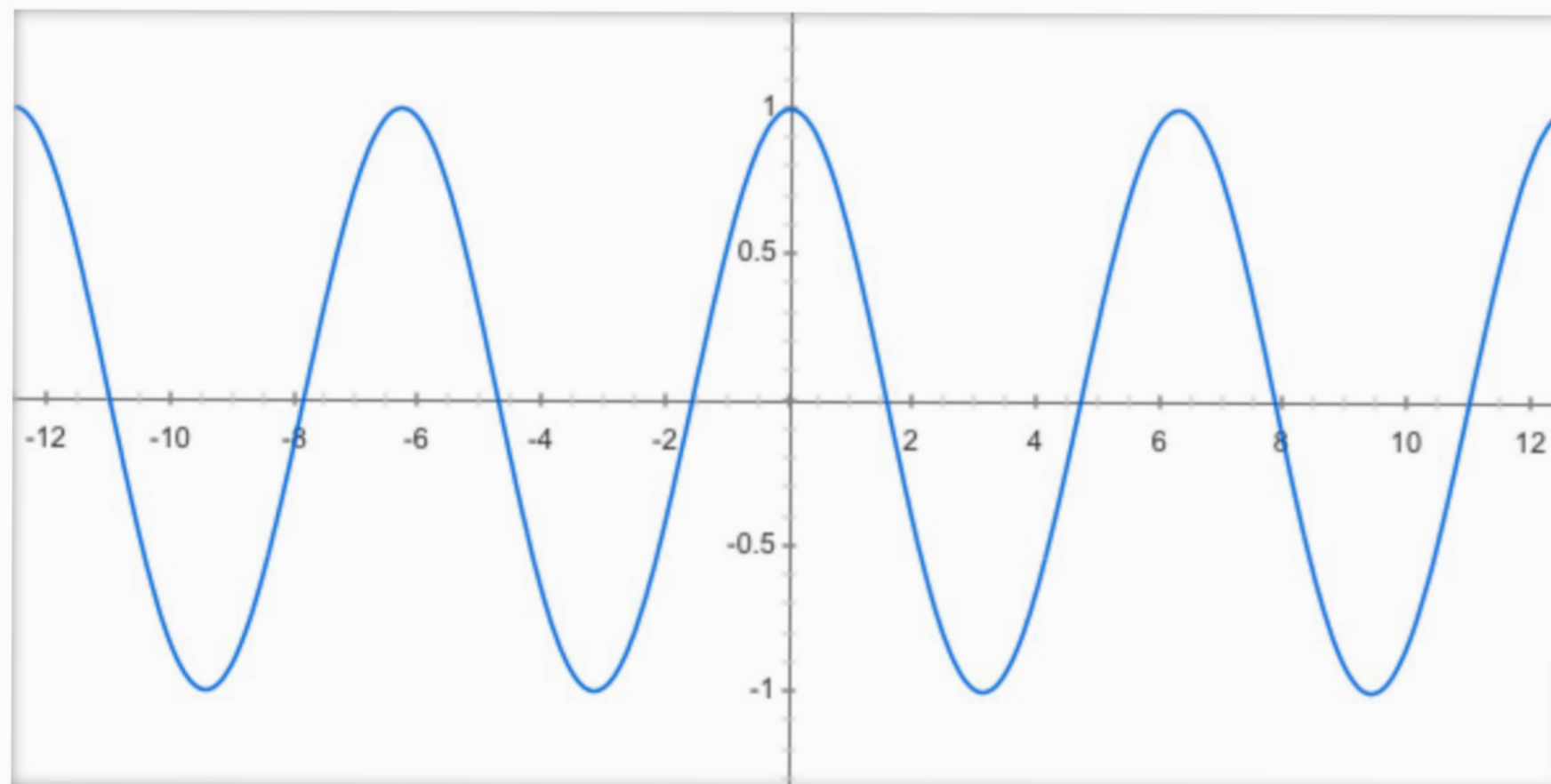


Even Functions: $f(-x) = f(x)$

Graph for x^2

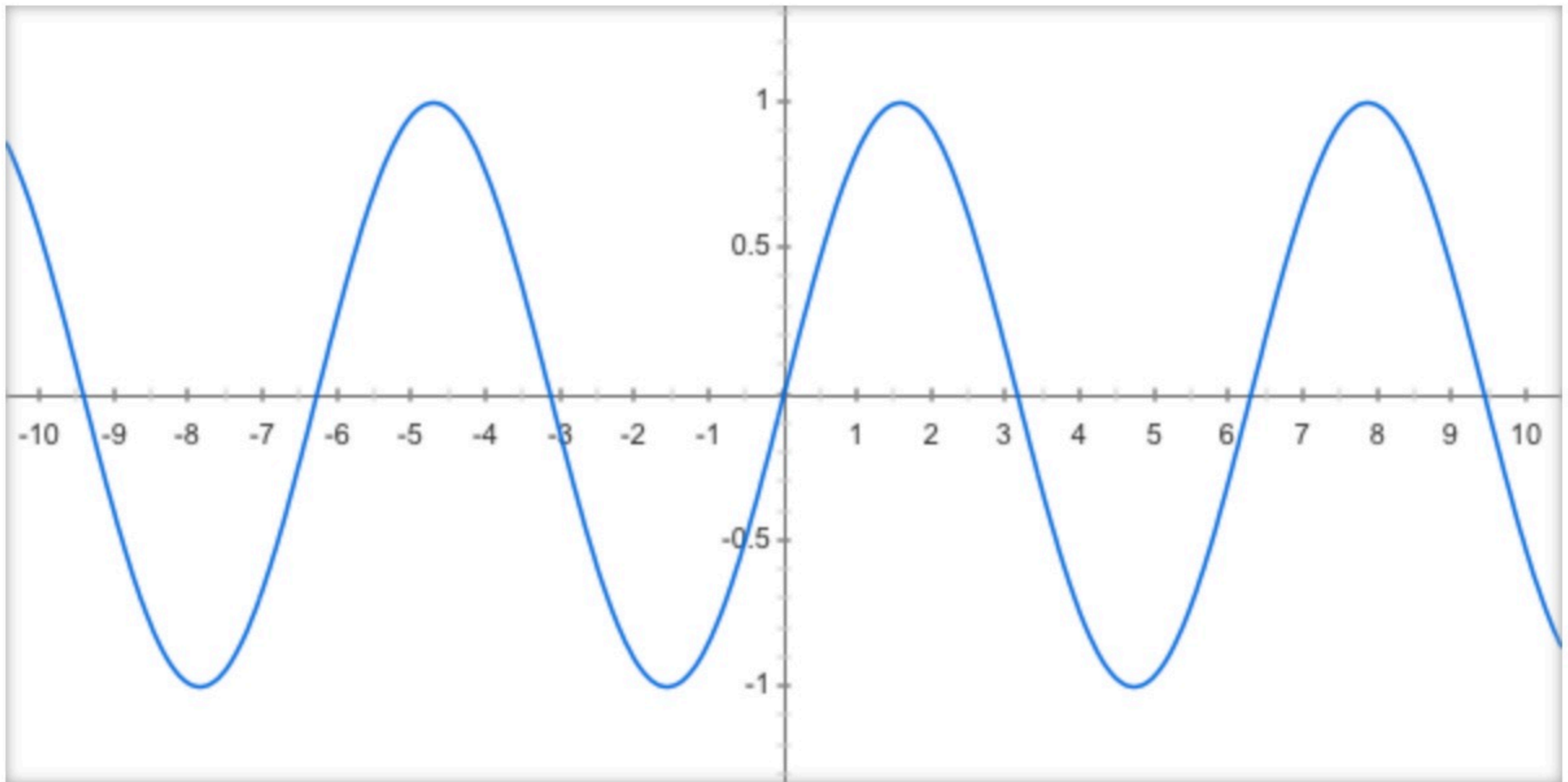


Graph for $\cos(x)$



Odd Function: $f(-x) = -f(x)$

Graph for $\sin(x)$



Example Sketch the graph of the function:

$$f(x) = \frac{x^2 - 3}{2x - 4}$$

Add information to graph
as you go!

y-intercept: $f(0) = \frac{-3}{-4} = \frac{3}{4}$

x-intercepts: $f(x) = 0 \Rightarrow \frac{x^2 - 3}{2x - 4} = 0 \Rightarrow x^2 - 3 = 0$
 $\Rightarrow x = \pm\sqrt{3}$

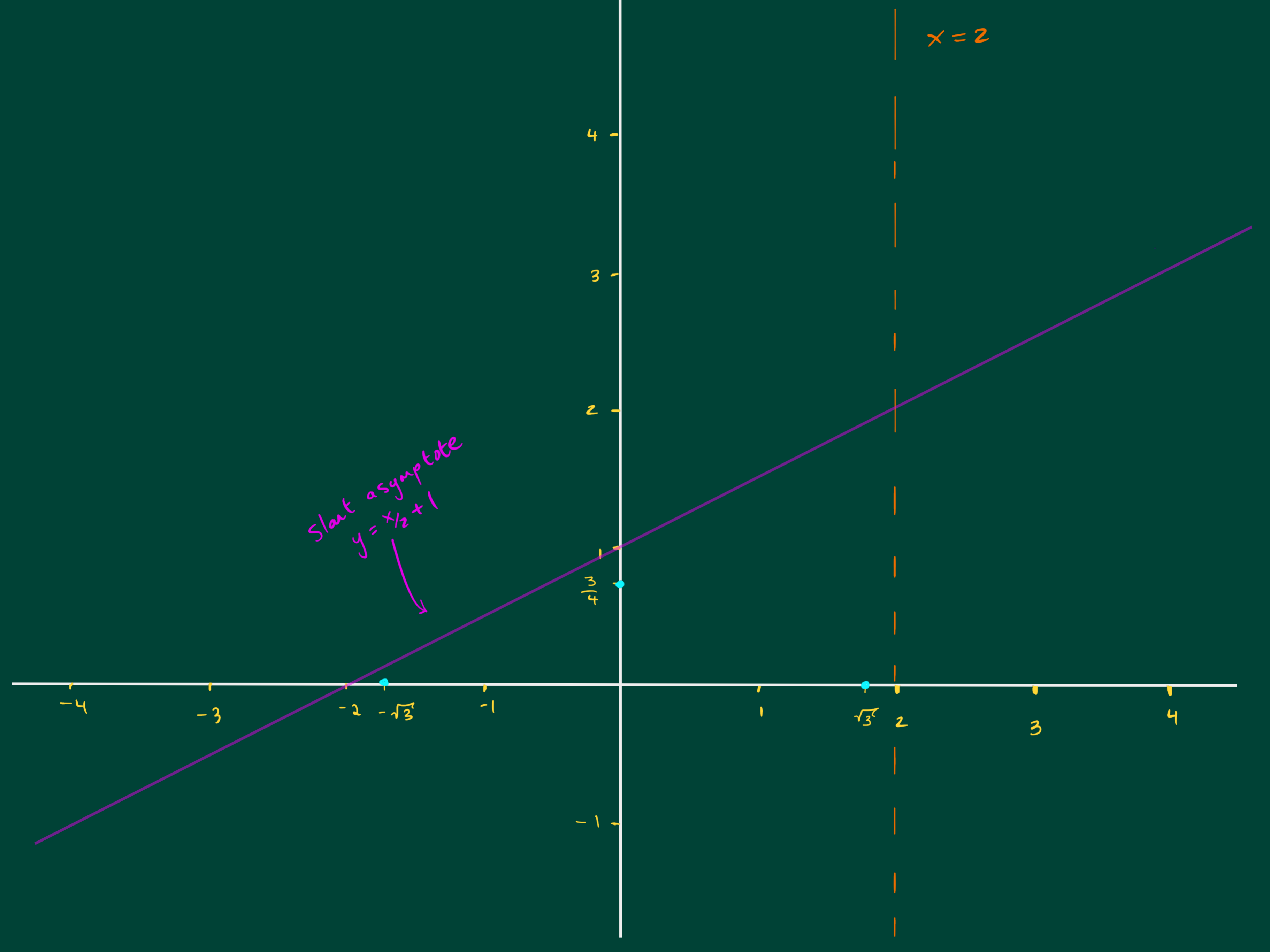
Asymptotes: Slant Asymptote: $y = \frac{x}{2} + 1$

Vertical asymptote at $x = 2$:

$$\lim_{x \rightarrow 2^+} f(x) = \lim_{x \rightarrow 2^+} \frac{x^2 - 3}{2x - 4} = +\infty$$

(Note: Orange arrows point from the top of the fraction to the right and from the bottom to the left, both labeled with a plus sign.)

$$\lim_{x \rightarrow 2^-} f(x) = -\infty$$



$x = 2$

Slant asymptote
 $y = x/2 + 1$

-4

-3

-2

$-\sqrt{3}$

-1

1

$\sqrt{3}$

2

3

4

-1

$3/4$

4

3

2

Critical pts: $f(x) = \frac{x^2-3}{2x-4} = \frac{1}{2} \left(\frac{x^2-3}{x-2} \right)$

$$f'(x) = \frac{1}{2} \left[\frac{(x-2)(2x) - (x^2-3)(1)}{(x-2)^2} \right]$$

$$= \frac{1}{2} \left[\frac{2x^2 - 4x - x^2 + 3}{(x-2)^2} \right]$$

$$= \frac{1}{2} \left[\frac{x^2 - 4x + 3}{(x-2)^2} \right]$$

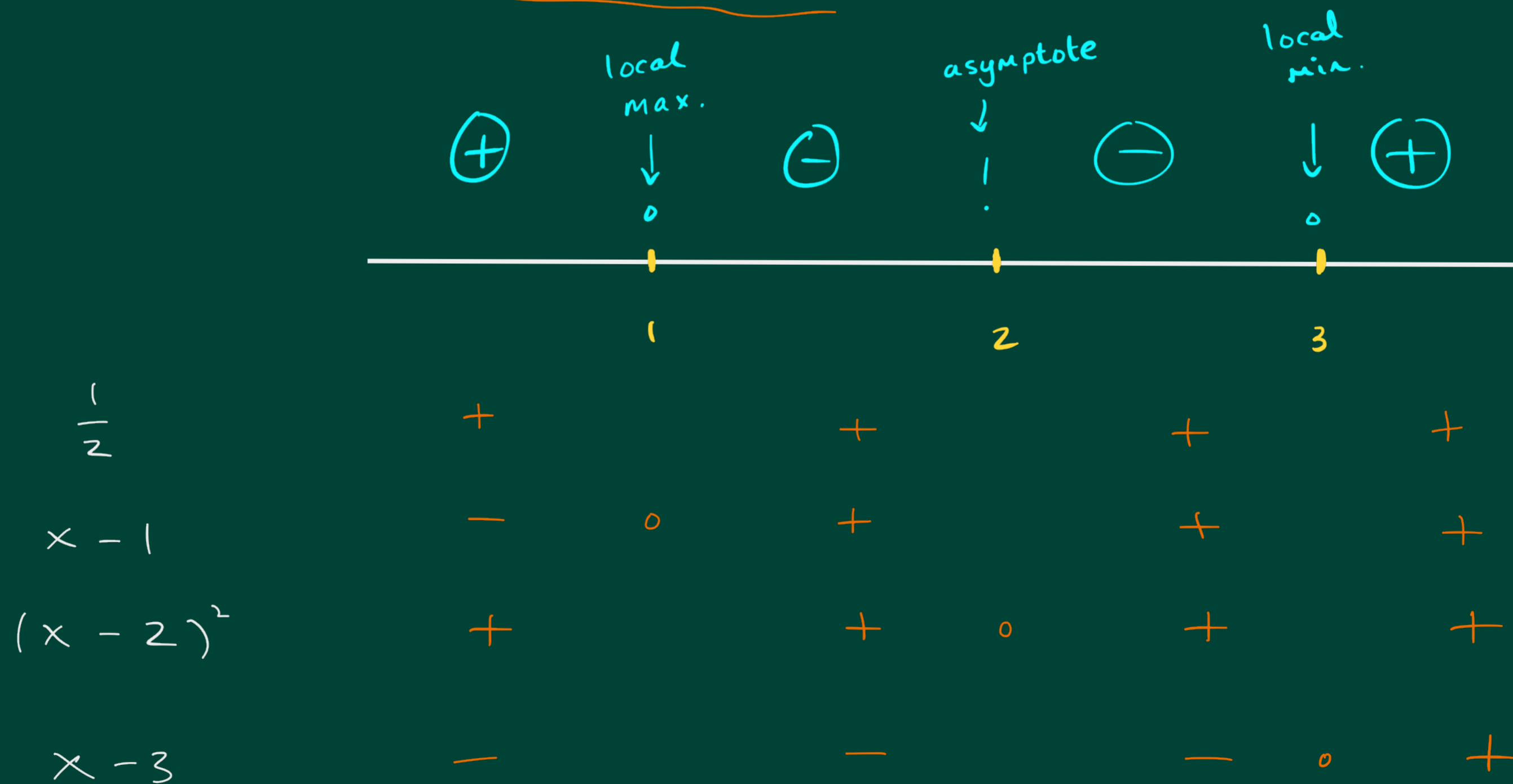
$$= \frac{1}{2} \left[\frac{(x-1)(x-3)}{(x-2)^2} \right]$$

Hence, critical pts at $x=1, 3$ and 2

$\underbrace{x=1, 3}$
 \uparrow
 $f'(1) = f'(3) = 0$

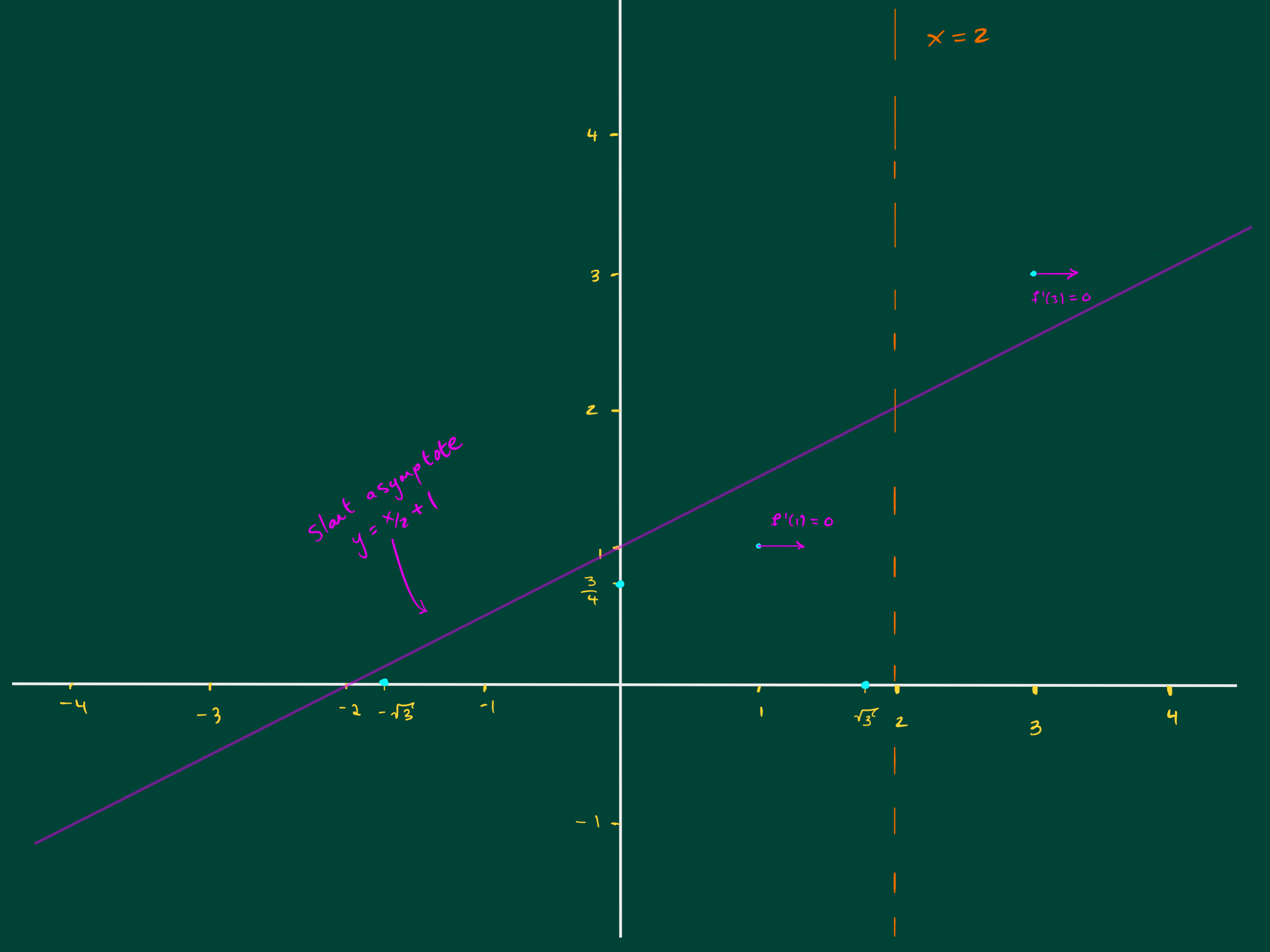
$\underbrace{2}$
 \uparrow
 $f'(2)$ undefined

Increasing / Decreasing \rightarrow Max. / Min. :



$$f(1) = \frac{(1)^2 - 3}{2(1) - 4} = \frac{-2}{-2} = 1 \quad : \quad (1, 1)$$

$$f(3) = \frac{(3)^2 - 3}{2(3) - 4} = \frac{6}{2} = 3 \quad : \quad (3, 3)$$



Concavity: $f'(x) = \frac{1}{2} \left(\frac{(x-1)(x-3)}{(x-2)^2} \right)$

$$f''(x) = \frac{1}{2} \left(\frac{(x-2)^2 \left((x-1)(1) + (x-3)(1) \right) - (x-1)(x-3)(2)(x-2)}{\left((x-2)^2 \right)^2} \right)$$

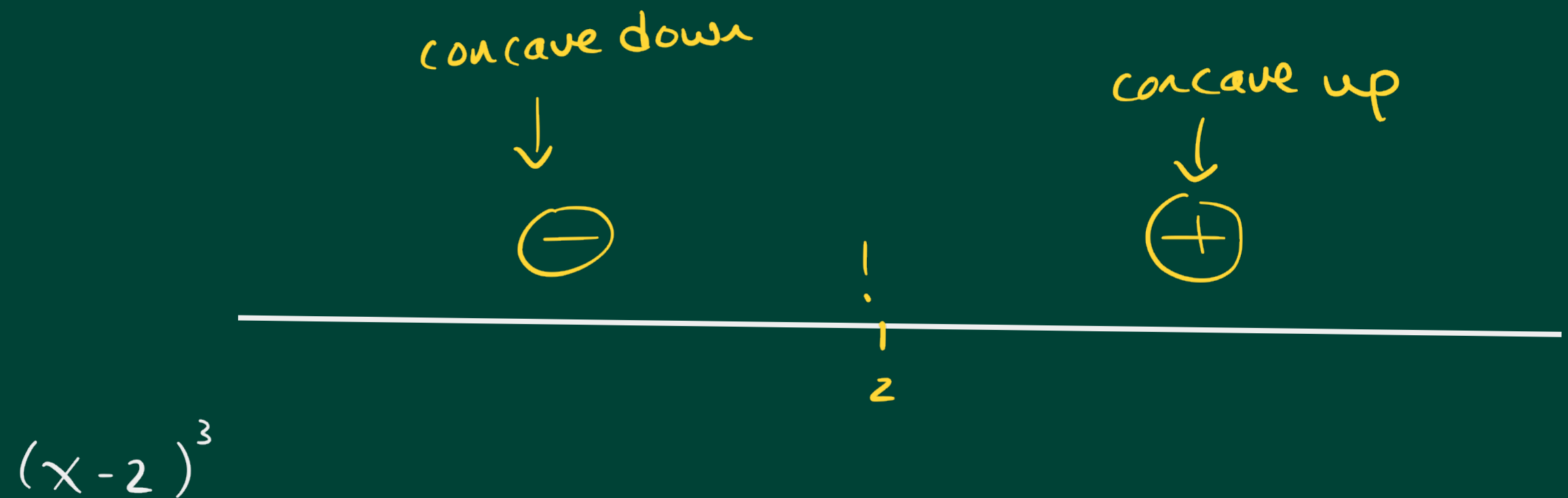
$$= \frac{1}{2} \left(\frac{(x-2)^2 (2x-4) - 2(x-1)(x-2)(x-3)}{(x-2)^4} \right)$$

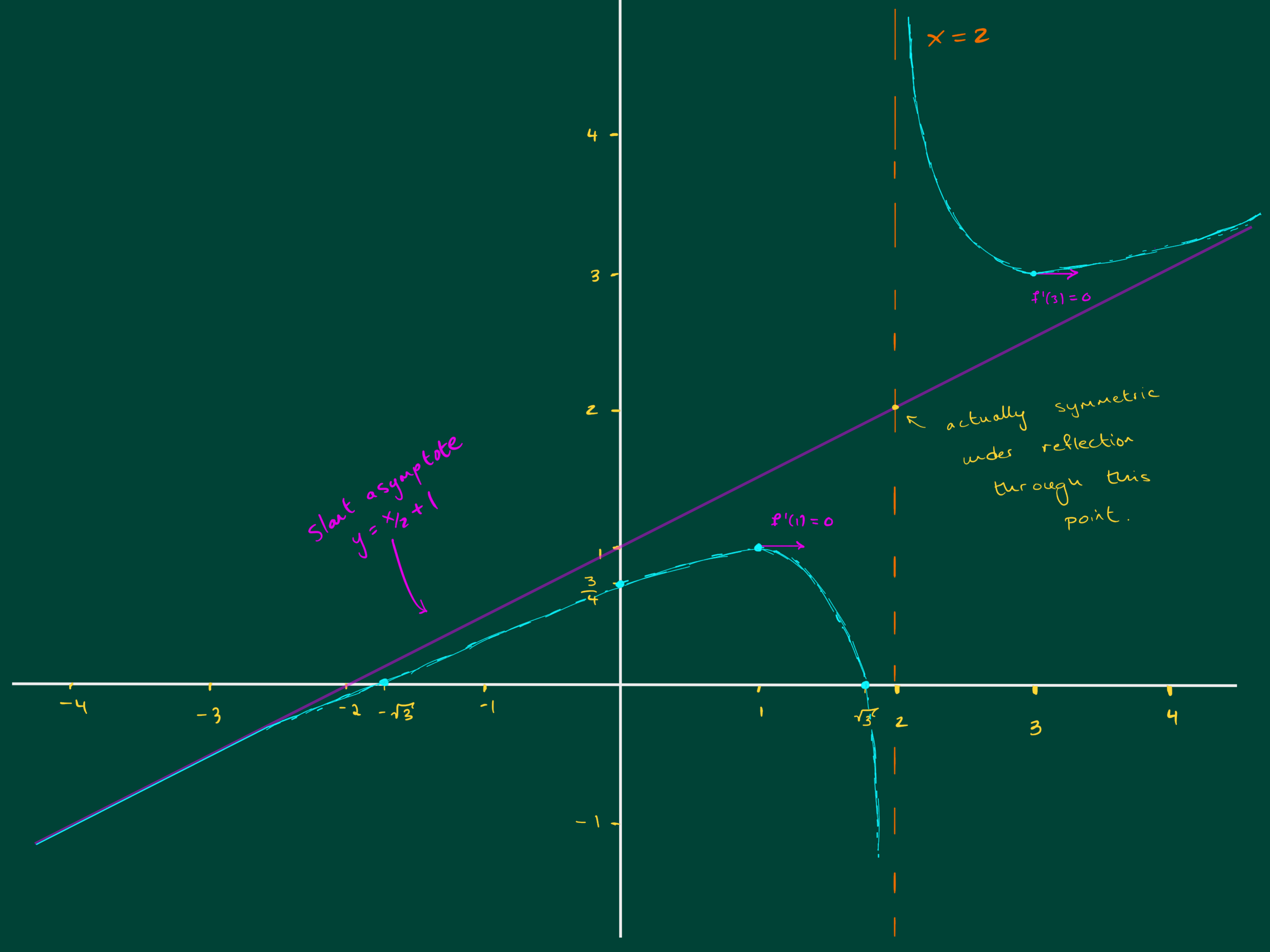
$$= \frac{1}{2} \left(\frac{\cancel{2}(x-2)^{\cancel{2}+2} - \cancel{2}(x-1)\cancel{(x-2)}(x-3)}{(x-2)^{4-3}} \right)$$

$$= \frac{(x-2)^2 - (x-1)(x-3)}{(x-2)^3}$$

$$= \frac{x^2 - 4x + 4 - x^2 + 4x - 3}{(x - 2)^3}$$

$$= \frac{1}{(x - 2)^3}$$





Computer generated graph:

Plot:

