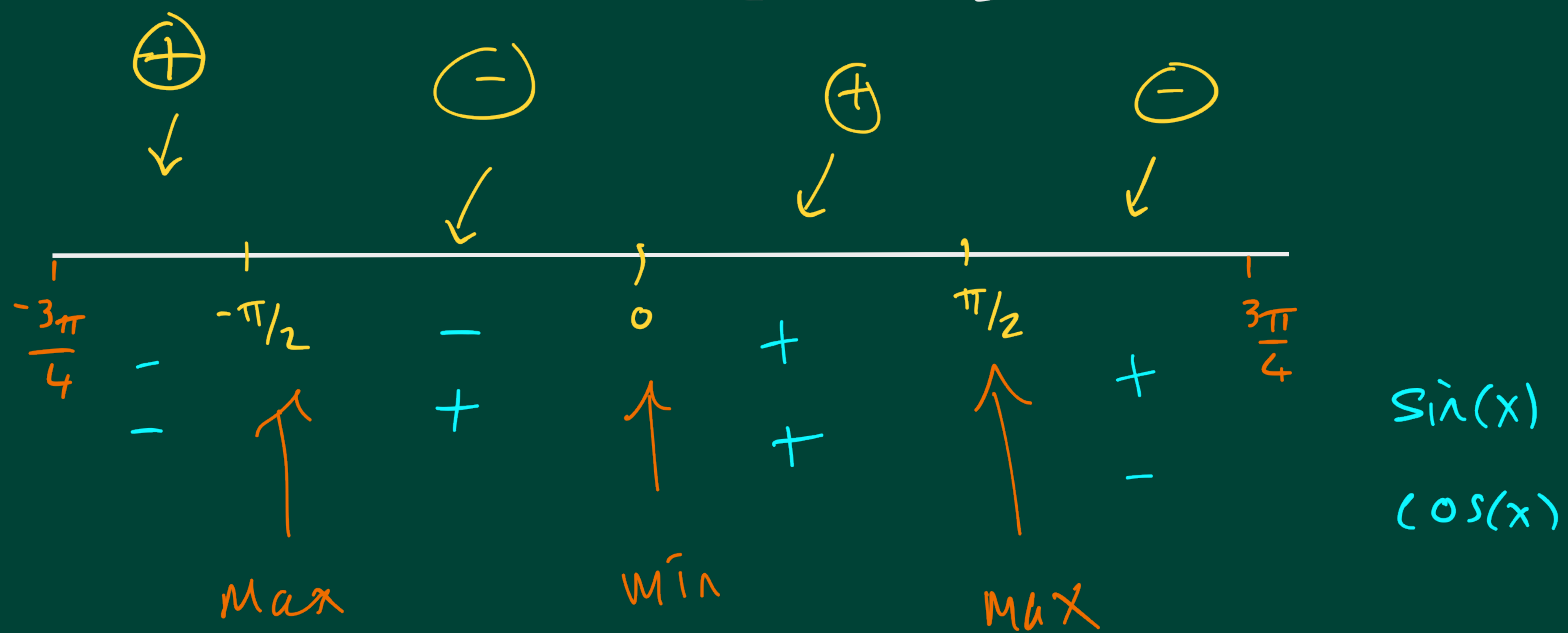
Ex:

$$g(x) = 1 - \cos^2 x \quad , \quad -\frac{3\pi}{4} \le x \le \frac{3\pi}{4}$$

$$g'(x) = 0 - 2\cos(x)\,(-\sin(x)) = 2\,\underline{\sin(x)}\;\underline{\cos(x)}$$

$$= \underline{\sin(2x)}$$

$$g'(x) = 0 \quad @ \quad x = 0 \;,\; \frac{\pi}{2} \;,\; -\frac{\pi}{2}$$



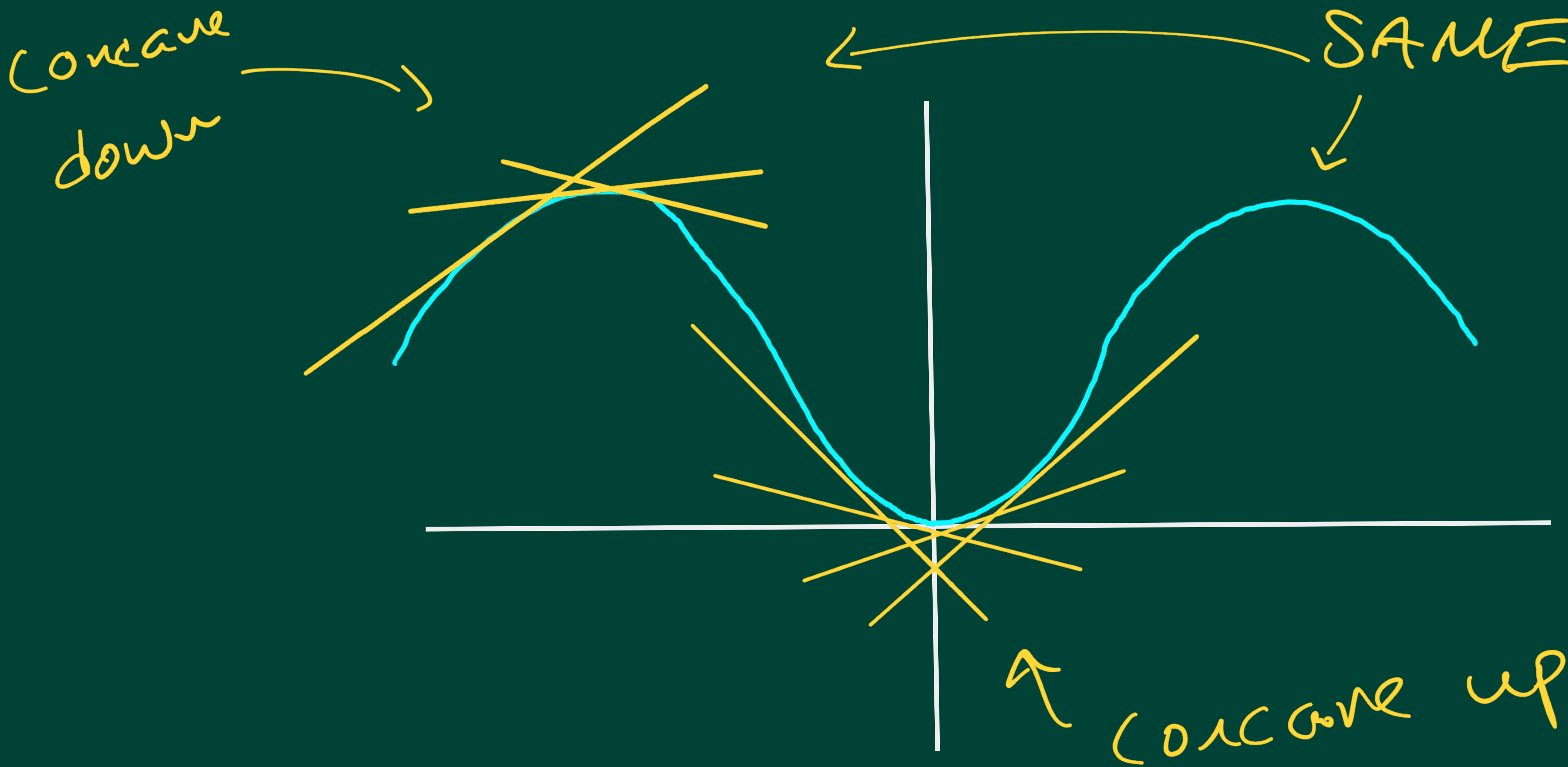
Without Calculus

$$g(x) = 1 - \cos^2 x = \sin^2 x$$

$$0 \le \sin^2 x \le 1$$

Max value is $g(x) = \sin^2 x = 1$ @ $x = \pm \frac{\pi}{2}$

Min value is $g(x) = 0$ @ $x = 0$



$$g''(x) = 2\cos(2x)$$

$$g''(x) > 0 \Rightarrow 2\cos(2x) > 0 \Rightarrow \cos(2x) > 0$$

$$\Rightarrow -\frac{\pi}{2} < 2x < \frac{\pi}{2} \Rightarrow \boxed{-\frac{\pi}{4} < x < \frac{\pi}{4}}$$

Exercise : Solve $g''(x) < 0$

and identify where graph is concave down.

Where are the pts of Inflection?

Step 1 : Solve $g''(x) = 0$

Step 2 : "Check for switch in sign."

$$2\cos(2x) = 0$$

$$\cos(2x) = 0$$

$$2x = \pm\frac{\pi}{2}$$

$$x = \pm\frac{\pi}{4}$$

Concave up : $\left(-\frac{\pi}{4}, \frac{\pi}{4}\right)$

Concave down : $\left(-\frac{3\pi}{4}, -\frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{3\pi}{4}\right)$

Hence, $\pm\frac{\pi}{4}$ are inflection pts.

Example:

$$f(x) = x^4 - 4x^3 + 10$$

(a) $f'(x) = 4x^3 - 12x^2$

$f'(x) = 0$

careful!

$4x^3 - 12x^2 = 0 \longrightarrow 4x - 12 = 0$

$x = 3$

$4x^2(x - 3) = 0$

$x^2 = 0$ or $\boxed{x = 3}$

$\boxed{x = 0}$

(b) $f''(x) = 12x^2 - 24x = 12x(x-2)$

$f''(0) = 0$ ← 2nd derivative test is inconclusive

$f''(3) = 108 - 72 = 36 > 0$

2nd derivative says local min.

$f'(x)$:	−	−	+
$4x^2$	+	+	+
$x-3$	−	−	+
	0	3	
	decreasing	decreasing	increasing

critical pts

$f''(x)$:	+	−	+	+
$12x$	−	+	+	+
$x-2$	−	−	+	+
	0	2	3	
	concave up	concave down	concave up	

$$f(x) = x^4 - 4x^3 + 10$$

points of interest

$$f(0) = 10 \qquad (0, 10)$$

$$f(2) = 2^4 - 4(2)^3 + 10 = -6 \qquad (2, -6)$$

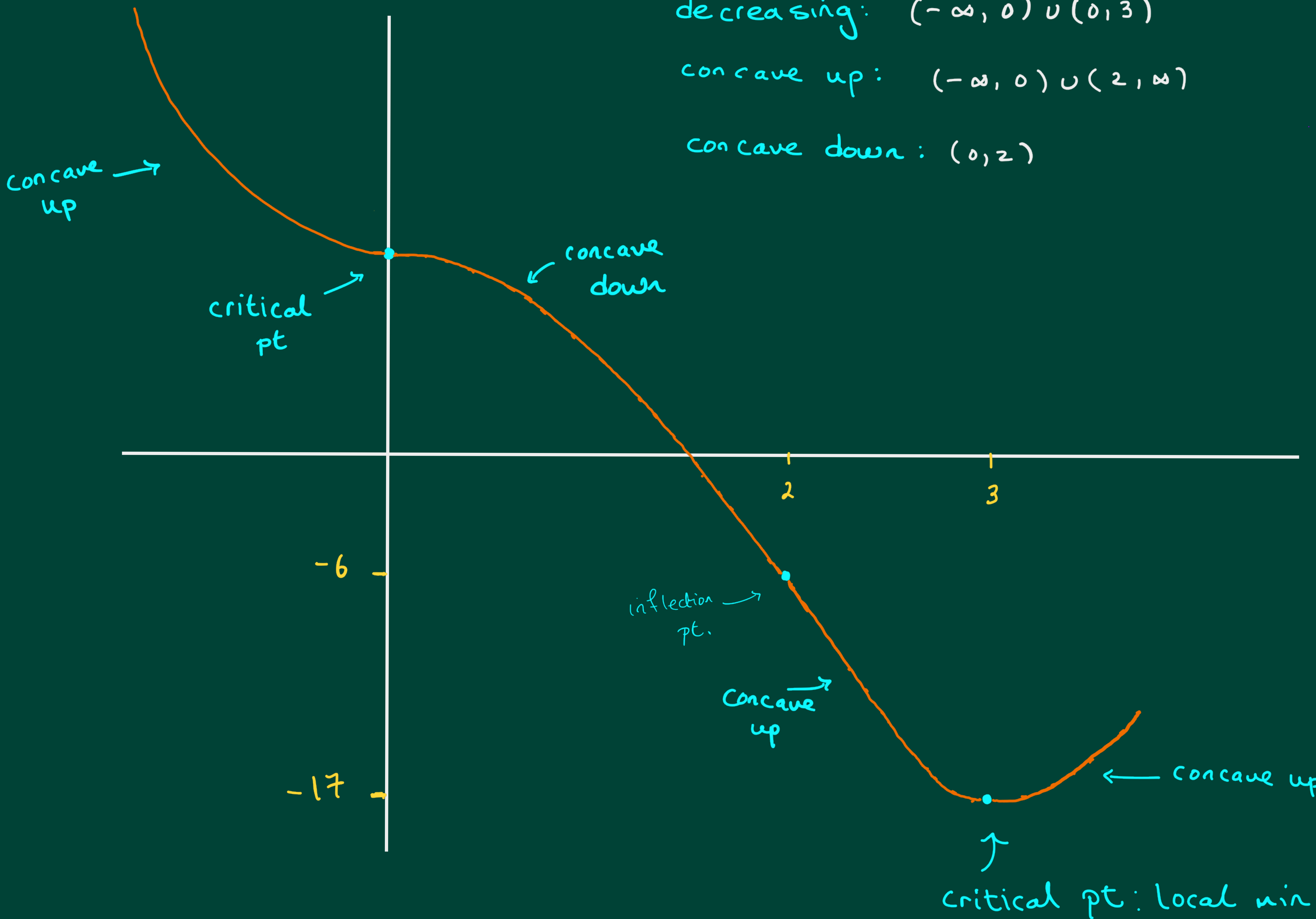
$$f(3) = 3^4 - 4(3)^3 + 10 = -17 \qquad (3, -17)$$

increasing: $(3, \infty)$

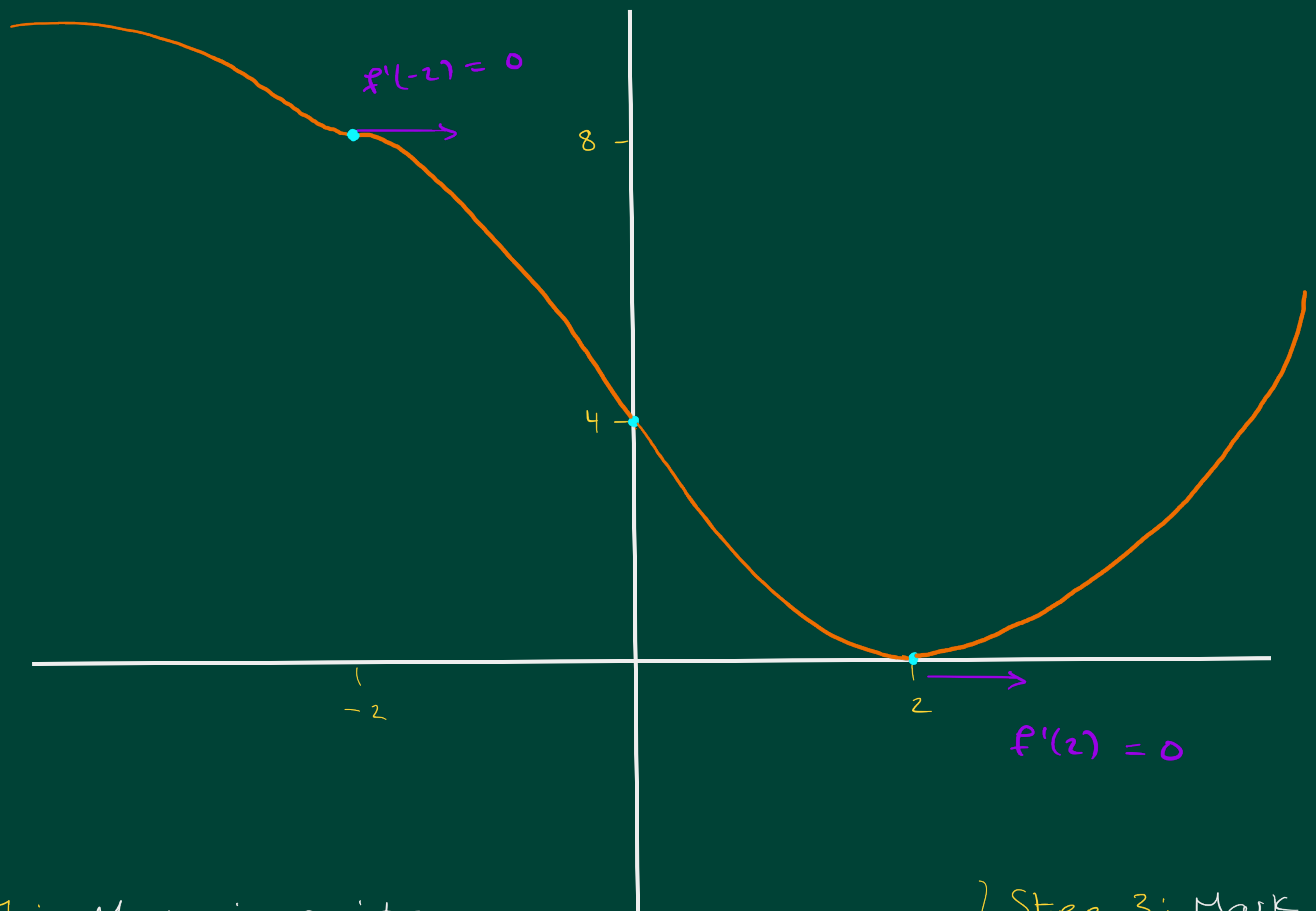
decreasing: $(-\infty, 0) \cup (0, 3)$

concave up: $(-\infty, 0) \cup (2, \infty)$

concave down: $(0, 2)$



Example: $f(-2) = 8$, $f(0) = 4$, $f(2) = 0$



Step 1: Mark in points.

Step 2: Mark in horizontal tangent lines

Step 3: Mark inflection pts.

Step 4: Connect pts with concave up/down curves.